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Sickness Recovery Intensities for Short Term Health Insurance in Greece

By

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Abstract

The last decennium a vast literature on multiple state models and stochastic models for disability (health) insurance has been developed. In our paper, with title sickness recovery intensities for short term health insurance in Greece, examines two methodologies for graduating sickness recovery intensities in a multistate model for short term health insurance. The approach is based on generalized linear models and utilizes the data collected for a Greek Social Security Institution. The sickness recovery intensity is function of age at sickness onset, x, and duration of sickness, z. The graduation formula that proposed for Greek experience has three break-point predictors.

JEL Classifications: C49, G22, I12.
Keywords: Disability Insurance, Average duration of a claim, Sickness recovery intensities, Graduation process.

1. Introduction

Sickness is one of the most serious threats to people's lives causing problems in his/her physical mental and economic situation. The economic loss is caused by a disease, concerns expenses for hospital and medical care and loss of income because of incapacity to continue to work for short or long time. To cover the risk for these economic loses many types of disability policies have been created by insurance companies and social security institutions. All these policies are under the general title health insurance. The term health insurance means any form of insurance whose payments is contingent on the insured incurring additional expenses because of the above risks (Black & Skipper, pp. 134-159). Health insurance classified into three categories, medical expenses, long term care insurance and loss of income (or disability income insurance). Particular disability income plans are classified into long-term and short-term according the time the insured received benefits.

A number of sources may potentially provide disability income benefits to employees (Rosenbloom, Victor, 1991, pp. 210-211):

1. Social security
2. Insurance company plans
3. Employer salary continuation plan
4. Other plans

Employees in Greece have been cover against sickness by the social security short term sickness policy. With the term short-term sickness benefits we mean disability benefits provided in the case of acute cases of sickness usually lasting no longer than same months. For every day out of work, after a three days deferred period, the employee receives a daily benefit, which may relate with employee's salary, duties in his work and the duration of sickness. The benefit is paid for short time usually six months. In this paper we use the term health insurance to describe short term disability of income policies.

To calculate the actuarial values of benefits have proposed various approaches. A first approach to the problem is well known as Manchester Unity model (Neill A., 1977, pp. 376-383). According to this approach selected tables for the sickness rate can by construct and actuarial values can be calculated using monetary functions. The definition for the sickness rate at age x:

$$z_x = \frac{\int_0^1 p_j(x+t).1_{x+t}.dt}{\int_0^1 1_{x+t}.dt} \quad (1)$$

where, $P_{ij}(x+t)$, is the probability the insured to be benefit against loss of income at age $x+t$ and 1_{x+t} the well known biometrical function which expresses the number of members who have survived at age $x+t$. Essentially, this approach relies on m-type estimation of benefit time.

At CMIR (1991) the Institute of Actuaries and the Faculty of Actuaries presented a more general approach for permanent disability insurance. This approach was based on Markov and semi-Markov stochastic processes. The use of Markov chains in life contingencies and their extensions has been proposed by several authors, for example Haberman (1999), Hoem (1969, 1988) and particular for disability benefits the mathematics of Markov and semi-Markov chains provide a powerful modeling tool. The CMIR model can be described as a three stage model. The stages were Health, Sick and Dead and the insured can move from one stage to another, except the stage Dead which was absorbing. The transition probabilities can be derived from transition intensities, which assigned from statistical data, via Kolmogorov differential equations. Based on CMIR model many papers were presented, for example Cordeiro (1998, 2002) which extend the model calculating the probabilities by cause of disability neither used different tools to calculate, Habermann & Ranshaw (1991, 1995, 2006), and Plat

(2009), Hatzopoulos (2009, 2011), Helwich (2008) works on the graduation of the transition intensities.

In this paper, first we present a multiple state model for short-term health insurance. The transition intensities are defined; Kolmogorov differential equations and formulae for the basic probabilities are presented. The main result of the paper is the graduation formula for sickness recovery intensities.

2. The model

The model proposed as a basis for the analysis of short-term health insurance data in Greece can be described with as a multiple state model with three states, Active (A), Sick (S), Other (D). On effecting his policy the policyholder enters state A and from this state he may transfer at any future time either to state S, become sick, or to state D, i.e. die or leave the policy or become permanent disability. Once in state S the policyholder transfer back to state A, i.e. recovery. All the probabilities in this model depend only on the policyholder's attained age except the recovery probability which depend on age and the duration in his current sickness. According Haberman and Pitacco (1999, p. 86) the policy conditions can be described as a vector $[n_1,n_2,f,m,r]$ where: (n_1,n_2) denotes insured period, f, is the deferred period, m, is the maximum time in years of annuity payment, r, is the stopping time of annuity payment.

For short term health insurance in Greece the vector is: $[c,\xi-x,d,m,\xi-x]$, where ξ is the retirement age.

The model can described more formally in terms of a pair of continuous time stochastic processes $\{S(x),R(x),x\geq 0\}$ where S(x) is the random state occupied by the risk at time x, S(x) takes values in the state space $F=\{A,S,D\}$, and R(x) is the time spent in state S(x) up to time x since the latest transition to that state, formally:

$$R(x)=\max\{\tau:\tau\leq t,S(t-h)=S(t),\ \forall h\in[0,r]\} \qquad (2)$$

R(t) takes values in [0,m]. Thus, the new stochastic process is defined by the pair of time-continuous stochastic processes {S(x), R(x)} it takes values in Fx [0,m]. Hence, Fx $[0,\infty]$ is the new state space.

Let us assume that $\{S(x), R(x),x\geq 0\}$ is time-continuous, time-inhomogeneous Markov process. This means that, at age x, all conditional probabilities concerning the future of the process depends only on the values of S(x) and R(x) and not on any other events prior to age x. If a policyholder has just fallen sick, the probability that he will remain sick for period z takes no account of information such as that he has experienced many periods of sickness in the past.

Following the notation from CMIR 12 (1991) we define the probability:

$$ {}_t p^{ij}_{x,z} = \Pr\{S(x+t) = j \mid S(x)=i \wedge R(z)=z\} \quad (3) $$

Where i,j = A,S,D and x,z,t ≥ 0.

The probabilities ${}_t p^{AS}_{x,z}, {}_t p^{AD}_{x,z}, {}_t p^{AA}_{x,z}$ are independent of the value of z and so we shall denote these probabilities ${}_t p^{AS}_{x}, {}_t p^{AD}_{x}, {}_t p^{AA}_{x}$ respectively. We also assume that ${}_t p^{DS}_{x} = {}_t p^{DA}_{x} = 0$ and ${}_t p^{DD}_{x} = 1$

The following probabilities can be calculated:

$$ {}_t p^{SA}_{x,z} = \Pr\{S(x+t) = A \,/\, S(x)= S \wedge R(x) \le z \le m\} \quad (4) $$

$$ {}_t p^{AS}_{x} = \Pr\{S(x+t) = S \,/\, S(x) = A\} \quad (5) $$

$$ {}_{w,t} p^{AS}_{x} = \Pr\{S(x+t) = S \wedge R(x+t) = w \,/\, S(x) = A\}, w < m \quad (6) $$

$$ {}_t p^{AD}_{x} = \Pr\{S(x+t) = D \,/\, S(x) = A\} \quad (7) $$

$$ {}_t p^{\overline{SS}}_{x,z} = \Pr\{S(x+t) = S \wedge R(x+t) = z+t < m \,/\, S(x) = S \wedge R(x) = z < m\} \quad (8) $$

$$ {}_t p^{\overline{AA}}_{x} = \Pr\{S(x+t) = A \wedge R(x+t) \ge t \,/\, S(x) = A\} \quad (9) $$

The transition intensities between the three states are denoted $r_{x,z}, \sigma_x, \mu_x$ and are defined as follows:

$$ r_{x,z} = \lim_{t \to 0+} \frac{{}_t p^{SA}_{xz}}{t} \quad (10) $$

$$ \sigma_x = \lim_{t \to 0+} \frac{{}_t p^{AS}_{x}}{t} \quad (11) $$

$$ \mu_x = \lim_{t \to 0+} \frac{{}_t p^{AD}_{x}}{t} \quad (12) $$

Transition intensity approaches (TIA) it is assumed that the transition intensities are assigned. From the intensities, via differential equations, the transition can in principle be derived. In the actuarial practice of insurances of the person, the intensities should be estimated from statistical data concerning recovery, disability and mortality.

Within the model, it is possible to derive the following differential equations:

$$\frac{\partial}{\partial t}\,{}_tp_x^{\overline{AA}} = -{}_tp_x^{\overline{AA}} \cdot \sigma_{x+t} \tag{13a}$$

$$\frac{\partial}{\partial t}\,{}_tp_{x,z}^{\overline{SS}} = -{}_tp_{x,z}^{\overline{SS}} \cdot r_{x+t,\,z+t} \tag{13b}$$

$$\frac{\partial}{\partial t}\,{}_tp_x^{AA} = -{}_tp_x^{AA} \cdot (\mu_{x+t,\,z+t} + \sigma_{x+t}) + \int_{u=0}^{t} {}_up_x^{AA} \cdot \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{SS} \cdot r_{x+t,t-u} \cdot du \tag{13c}$$

$$\frac{\partial}{\partial t}\,{}_{w,t}p_x^{AS} = \begin{cases} {}_{t-w}p_x^{HH} \cdot \sigma_{x+t-w} \cdot {}_wp_{x+t-w}^{SS}, & \text{if } 0 \le w \le t \\ 0, & \text{if } w > t \end{cases} \tag{13d}$$

$$\frac{\partial}{\partial t}\,{}_tp_x^{HD} = {}_tp_x^{HH} \cdot \mu_{x+t} \tag{13e}$$

All the above formulae can be regarded as Kolmogorov forward equations for the proposed model.

Integrating the above equations (13a-e), we obtain the following formulae:

$$\frac{\partial}{\partial t}\,{}_tp_x^{\overline{AA}} = -\,{}_tp_x^{\overline{AA}} \cdot \sigma_{x+t} \Rightarrow {}_tp_x^{\overline{AA}} = e^{-\int_0^t \sigma_{x+u} du} \tag{14}$$

$$\frac{\partial}{\partial t}\,{}_tp_{x,z}^{\overline{SS}} = -\,{}_tp_{x,z}^{\overline{SS}} \cdot r_{x+u,z+u} \Rightarrow {}_tp_{x,z}^{\overline{SS}} = e^{-\int_0^t r_{x+u,z+u}\,du} \text{ with } z + \le m \tag{15}$$

3. Importance of the model

As we have mentioned in previous section the model we proposed can be used to calculate actuarial values and quantities such as the average duration of a claim. This is necessary for pricing health insurance programs and also in the actuarial valuation of Social Health Programs. The total cost of a disease can be divided into direct and indirect cost.

The direct cost includes the cost of medical procedures, cost for possible hospital admission, as well as pharmaceutical cost. Greece since 2011 has begun to create Greek methodology for Closed Consolidated Medical Bills (KEN). The KEN constitutes an attempt to establish the Greek version of DRGs (GR-DRGs), i.e. the introduction of prospective funding and reimbursement of health services to the Hospitals (Polizos, 2008). Indirect costs relate to the costs incurred by the insured's absence from work, loss of revenue of contributions and the concurrent increase in expenses due to subsidization of days out of work. The results of this study are important in estimating the indirect costs.

It is well known (Haberman & Pitacco, 1999, pp. 11-13) that actuarial evaluations, which are needed to calculate premiums and mathematical reserves, include the calculation of present and expected values. To perform calculation of present and expected values we need a financial and a probabilistic structure. The simplest financial structure is compound interest with constant force of interest δ and the discount function is equal to e^{-dt}.

Let $b_s(t)$ the benefit paid to the policyholder if $S(t)=S$ and $d \leq R(t) \leq m$ hence $b_s(t) \cdot dt$ is the benefit paid to time interval $[t, t+dt]$. Let $I_{\{S(t)=c\}}$ the indicator of the event $S(t) = S$ and $d \leq R(t) \leq m$, i.e.:

$$I_{\{S(t)=c\}} = \begin{cases} 1, & \text{if } S(t)=S \text{ and } d \leq R(t) \leq m \\ 0, & \text{otherwise} \end{cases}$$

Then the random present value at time t_0 is:

$$Y_{t_0}(t) = u^{t-t_0} \cdot I_{\{S(t)=c\}} \cdot b_s(t) \cdot dt \tag{16}$$

and the random present value of the annuity benefit on the time interval (u_1, u_2) is:

$$Y_{t_0}(u_1,u_2) = \int_{u_1}^{u_2} Y_{t_0}(t) \cdot dt = \int_{u_1}^{u_2} u^{t-t_0} \cdot I_{\{S(t)=c\}} \cdot b_s(t) \cdot dt \tag{17}$$

Actuarial values are the expected present so for equations (16) and (17) we take:

$$E(Y_{t_0}(t) \mid S(t_0) = A) = u^{t-t_0} \cdot {}_tp_x^{AS} \cdot b_s(t) \tag{18}$$

and

$$E(Y_{t_0}(u_1,u_2) \mid S(t_0) = A) = \int_{u_1}^{u_2} u^{t-t_0} \cdot {}_tp_x^{AS} \cdot b_s(t) \cdot dt \tag{19}$$

Considering constant benefit $b_s(t)=b$ and under the UDD hypothesis, the following approximation can be assumed:

$$E(Y_{t_0}(u_1,u_2)/ S(t_0) = A) \approx \sum_{h=u_1}^{u_2} u^{h-\frac{1}{2}-t_0} \cdot b \cdot \int_{h-1}^{h} {}_tp_x^{AS} \cdot dt \tag{20}$$

If we denote by T_x the duration of a claim, for a claimant aged x at the beginning of the corresponding sickness, the average duration of this claim (in years) is given by (Cordeiro, 2002, p. 175):

$$E(T_x) = \frac{\int_d^m t \cdot {}_tp_x^{\overline{SS}} \cdot r_{x+t,t} dt}{\int_d^m {}_tp_x^{\overline{SS}} \cdot r_{x+t,t} dt} \tag{21}$$

From equations (20) and (21) it is obvious that we need $r_{x+t,t}$ to calculate average duration of claim actuarial values.

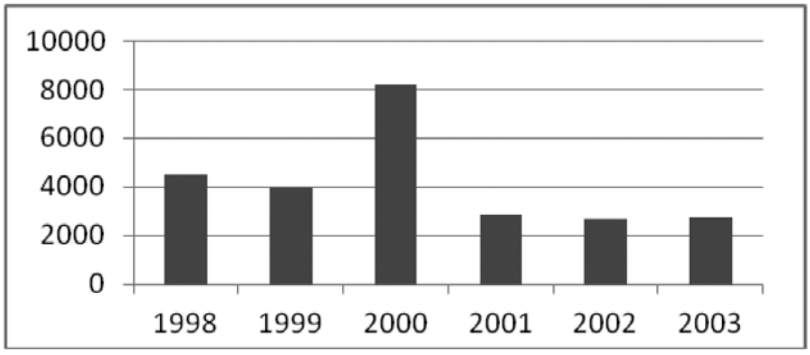
4. The data

The data come from a Social Security Agency concerning claims for the years 1998 to 2003. Unfortunately, Social Security Organizations in Greece haven't organized yet, a system for collecting data about the duration of the disability.

The distribution of cases per year is shown in Figure 1. Each claimant is classified according age and sickness duration. For modeling purpose, the data are located at the centroid of their respective cells determined by weighted averaged, with relative exposure as weights.

FIGURE 1

Distribution of claims per year



Data is classified into nine age groups: 20-24, 25-29,...,60-64. Table 1 presents the mean age, calculated as mention above, for each age group.

Data is classified into 10 sickness period: 0-1 week, 1-2 weeks,...,9-10 weeks and the centre of the interval is calculated (Table 2). For modeling purpose the duration converted from days to years assuming that one year equals 365.25 days.

5. Sickness recovery intensities

To estimate the values $r_{x,z}$ we follow the methodology presented in CMIR 12. This methodology can briefly described

- We choose age intervals $x_1 \leq x \leq x_2$ and sickness period $z_1 \leq z \leq z_2$ which are sufficiently small for us to accept that $r_{x,z}$ is approximately constant over the rectangle $[x_1,x_2]$ x $[z_1,z_2]$.

• We calculate the number of sickness recoveries, $O_{x,z}$, over the rectangle $[x_1, x_2]$ x $[z_1, z_2]$.

TABLE 1

Mean age by age group

Age group	Mean age
20-24	22.88
25-29	27.17
30-34	32.02
35-39	36.96
40-44	41.91
45-49	47.01
50-54	52.03
55-59	56.75
60-64	61.40

• We calculate the total time, $E_{x,z}$, spent sick in the observation period, counting only the time when the policyholder were aged between x_1 and x_2 and when the duration of their sickness was between z_1 and z_2.

TABLE 2

Average sickness duration (days) by sickness period

Sickness period	Centre of interval (days)
1 week	5.6
2 weeks	10.4
3 weeks	16.71
4 weeks	24.55
5 weeks	32.12
6 weeks	38.69
7 weeks	46.01
8 weeks	52.85
9 weeks	60.53
10 weeks	65.4

Under reasonable assumptions (CMIR 12, 1991, pp. 10-12) the maximum likelihood estimator of $r_{x,z}$ is $\widehat{r_{x,z}}$ where:

$$\widehat{r_{x,z}} = \frac{o_{x,z}}{E_{x,z}} \tag{22}$$

For large samples sizes, Schou & Vaeth (1980) suggest this assumption is reasonable if expected number of recoveries exceeds 10, we assume that:

$$\widehat{r_{x,z}} \sim N\left(r_{x,z}, \frac{o_{x,z}}{E_{x,z}^2}\right) \tag{23}$$

Table 3 presents the crude sickness recovery intensities.

6. The graduation process

Graduation may be regarded as the principles and methods by which a set of crude values is adjusted to provide a suitable basis for inferences to be made and further practical calculation to be made (Haberman & Renshaw, 1996, p. 411).

There are many graduation methods that have been suggested. We can classify these methods to three categories graphic methods, parametric methods and non parametric methods. In this paper, we present a parametric approach to graduation based on generalized linear models.

Sickness recovery intensities $r_{x,z}$ are perceived as functions of two covariates, age x and sickness duration z, so the graduation process is like an exercise in surface fitting.

TABLE 3

Crude sickness recovery intensities

Age Group	Duration (weeks)									
	1	2	3	4	5	6	7	8	9	10
20-24	9.6893	23.9363	40.1923	14.1084	17.8171	9.2671	19.5888	9.0046	10.3961	14.6791
25-29	9.9617	28.7064	41.6407	18.2863	20.4584	12.2276	13.2048	10.2887	12.3814	10.6531
30-34	9.3368	30.7037	52.4687	14.4024	18.6552	8.4009	14.3097	9.1980	9.0615	9.6004
35-39	8.6342	26.0488	44.3017	14.8971	20.5197	10.7602	13.6048	10.8347	10.6989	21.2850
40-44	7.7638	26.9292	43.8871	14.3191	21.3932	8.3240	12.8401	10.9225	15.2528	17.9765
45-49	6.7798	25.1078	40.7328	13.9724	21.3266	12.1333	9.6316	7.2759	8.6572	20.3244
50-54	6.5576	21.3218	36.0964	12.7897	17.4731	9.5817	9.7734	7.8852	14.4969	20.9199
55-59	5.1422	21.0652	32.2751	14.3534	14.5416	6.9180	8.7202	8.6715	12.6873	13.6118
60-64	6.2526	15.8071	22.2554	7.7202	17.5722	10.9339	5.1540	7.6976	11.7531	18.5829

The deferred period is three days and a notable feature of the sickness recovery data is the low recovery rate associated with the week immediately after the sickness benefit becomes payable (1st week). A possible reason for this is because the policyholders near to recovery at the end of deferred period are less likely to bother to submit a claim on the grounds that the claim would be short-lived. We also note an irregular recovery rate change between six and eight weeks.

Let $Y=(Y_t)$ a vector of independent random variables $Y_t, i=1,2...,n$ and $m=E(Y)$ a vector of means. GLM is characterized by three basic ingredients:

- The modeling distribution of random variable Y_t,
- Covariates x_t and the linear predictor

$$n_{x,z} = \sum_j h_j(x,z) \cdot b_j$$

- Link-function g, which link the linear predictor with means m

$n_{x,z} = g$ (m).

As a general rule the modeling distributions available for use are restricted to the exponential family of distributions (Renshaw, 1991, p. 295).

The function g is both differentiable and one to one so that, the inverse function g^{-1} exist and $m = g^{-1}(n_{x,z})$

In our case we have: $g(r_{x,z})=n_{x,z} \Rightarrow r_{x,z}=g^{-1}(n_{x,z}) \Rightarrow r_{x,z}=g^{-1}(\Sigma_j f_j(x,z) \cdot b_j)$.

Estimates of the unknown b_j and diagnostic checks based on the assumption that $O_{x,z}$ follow the Poisson distribution, $O_{x,z} \sim P(E_{x,z} \cdot r_{x,z})$

with mean and variance respectively $m_{x,z}=E(O_{x,z})=E_{x,z} \cdot r_{x,z}$ and $Var(O_{x,z})=m_{x,z}$.

From the above relations we have $m_{x,z}=E_{x,z} \cdot r_{x,z}=E_{x,z} \cdot g^{-1}(\Sigma_j f_j(x,z) \cdot b_j)$.

The estimations, $\widehat{b_j}$, for the unknown parameters b_j, are calculated by maximizing the log-likelihood. Denoting the resulting fitted values by $\widehat{m_{x,z}} = E_{x,z} \cdot g^{-1}(\sum_j f_j(x,z) \cdot \widehat{b_j})$ the optimum value for the log-likelihood under the current model structure, c, is:

$$\ln(l_c) = \ln\left(l(O;\widehat{m})\right) = \sum_u \{-\widehat{m}_u + O_u \cdot \ln(\widehat{m}_u)\} + c$$

Let f be the saturated model, which has the property that its fitted values, $\widehat{m_u} = O_u$ constitute a perfect fit. The log-likelihood under the saturated model

becomes: $\ln(l_f) = \ln\left(l(O;O)\right) = \sum_u \{-O_u + O_u \cdot \ln(O_u)\} + c$

and the model deviance $D(c, f)$ is

$$D(c,f) = \sum_u d_u = -2 \cdot \ln\left(\frac{l_c}{l_f}\right) = -2 \cdot \ln(l_c) + 2 \cdot \ln(l_f) = 2 \cdot \sum_u \left\{-(O_u - \hat{m}_u) + O_u \cdot \ln\left(\frac{l_u}{\hat{m}_u}\right)\right\}$$

The overall measure of goodness of fit is provided by the model deviance. Another criterion for evaluate the goodness of fit is Bayes Information Criterion (BIC). The BIC is defined like: $BIC = L(\hat{m}) - \frac{1}{2} \cdot K \cdot \ln N$

where K is the number of parameters and N is the number of observation (Plat, 2009).

The reader is referred to McCullagh and Nelder (1989, pp. 357-371) for further commentary on diagnostic model checking.

A mentioned above sickness recovery intensities $r_{x,z}$ are perceived as functions of two covariates, age x and sickness duration z and the liner predictor is assigned the additive structure

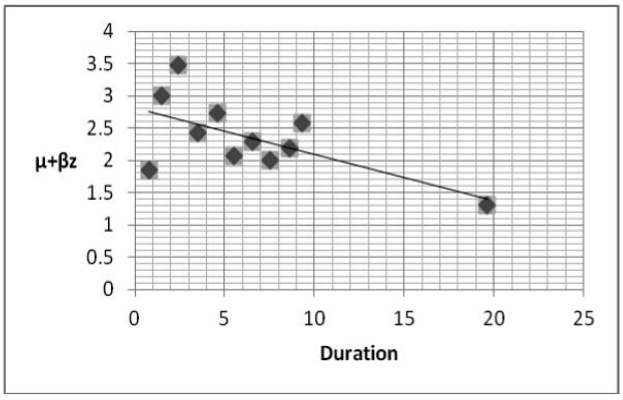
$$n_{x,z} = \mu + \alpha_x + \beta_z \quad (24)$$

And under the log link the graduation formula is $r_{x,z} = e^{(\mu + \alpha_x + \beta_z)}$.

Figure 2 and 3 reproduce the graphs of parameter estimates $\hat{\mu} + \hat{\beta}_z$ against duration z and parameter estimates $\hat{\alpha}_x$ against age x. We note that apart from some fluctuations α_x, β_z appears to reduce linearly with x and z respectively.

FIGURE 2

Sickness parameters against duration



According the notes for Fig 2 and 3 we approached the graduation using log link and a range of possible linear predictors of the two varieties x and z. We use two kind of linear predictors, fractional polynomials and break- point predictor terms. The second approach used to model the effects noted in section 6.

Fractional polynomial of degree m is the function

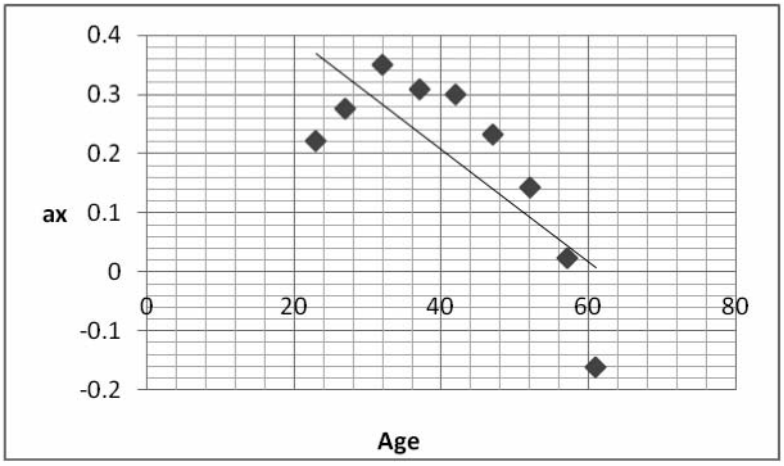
$$\varphi_m(X,\xi,p)=\xi_0+\sum_{\xi=1}^{m}\xi_j\cdot X^{(p_j)}$$

where X is a single covariance, m is a positive integer, $p=(p_1,p_2,...,p_m)$ is a real value vector of powers with $p_1<p_2<...<p_m$ and $\xi=(\xi_0,\xi_1,...,\xi_m)$ are a real- valued coefficients. The round bracket notation signifies the Box-Tidwell transformation,

$$X^{(p_j)}=\begin{cases}X^{p_j} & \text{if } p_j\neq 0\\ lnX & \text{if } p_j=0\end{cases}.$$

FIGURE 3

Age at sickness parameters against age



Royston and Altman (1994, pp. 429-436) extended the definition for any arbitrary powers $p_1\leq p_2\leq...\leq p_m$, with $H_0(X)=1$ and $p_0=0$,

$$\varphi_m(X,\xi,p)=\sum_{\xi=0}^{m}\xi_j\cdot H_j(X)$$

where for $j=1,...,m$

$$H_j(X)=\begin{cases} X^{(p_j)} & if\ p_j \neq p_{j-1} \\ H_{j-1}(X)\cdot lnX & if\ p_j = p_{j-1} \end{cases}$$

They used the notation $\varphi_m(X;p)$ to express the equations, for example $\varphi_m(X;0,1,2,2,2)$ means the equation with components: $H_0=1$, $H_1=lnX$, $H_2=X$, $H_3=X^2$, $H_4=X^2\cdot lnX$, $H_5=(X\cdot lnX)^2$. For modelling a data set of size n using fractional polynomials, they suggest to determine the best value of m and the real-valued vector $p=(p_1, p_2, ..., p_m)$ selected with replacement from the fixed set $F=\{-2, -1, -0.5, 0, 0.5, 1, 2, ..., \max(3,m)\}$. They also proposed a two step procedure to fit multiple covariates.

The graduation formula that is adopted is a combination of age x and duration z:

$$r_{x,z}=e^{(b_0+b_1\cdot x+b_2\cdot\sqrt{x}+b_3\cdot\sqrt{z}+b_4\cdot z+b_5\cdot z^2+b_6\cdot x^2+b_7\cdot x\cdot lnx)} \quad (25)$$

The parameters estimation presents in Table 4 and the goodness of fit checks to Table 6.

TABLE 4

Parameter estimation for fractional model

Linear predictors with break-point predictor terms are functions of the type:

Parameter	DF	Estimate	Standard Error	Likelihood Ratio 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
b_0	1	743.3933	7.3358	520.0233	967.5828	10269.3	<.0001
b_1	1	225.8771	0.3849	159.2621	292.7079	344464	<.0001
b_2	1	-655.144	3.1851	-850.157	-460.799	42307.4	<.0001
b_3	1	27.7082	3.6860	20.5499	35.0035	56.51	<.0001
b_4	1	-28.9083	8.2734	-45.2521	-12.8117	12.21	0.0005
b_5	1	-23.8867	9.7223	-42.8721	-4.7476	6.04	0.0140
b_6	1	0.1746	0.0016	0.1246	0.2247	11828.9	<.0001
b_7	0	-40.1140	0.0000	-51.9336	-28.3311	.	.

$$\sum_{j=0}^{J} b_{0j}\cdot z^j+\sum_{k=1}^{K}\sum_{j=1}^{J} b_{kj}\cdot(z-z_k)^j_+$$

with knots z_k, where $(z-z_k)^j_+=\begin{cases} z-z_k, & if z>z_k \\ 0, & otherwise \end{cases}$.

We propose a linear predictor with three knots, $z_1=0.019$, $z_2=0.038$, $z_3=0.057$

$$r_{x,z} = e^{(b_0+b_1\cdot x^2+b_2\cdot x+b_3\cdot z^2+b_4\sqrt{z}+b_5\cdot(z-z_1)_+ +b_6\cdot(z-z_2)_+ +b_7\cdot(z-z_2)_+)} \quad (26)$$

The parameters estimation presents in Table 5 and the goodness of fit checks to Table 6.

To comparison fit quality for the two approaches we use deviance and Bayes Information Criterion. The results present at Table 6. The table shows that for Greek experience the break- point model gives the best fitting results. The predicted values present at Table 7.

TABLE 5

Parameter estimation for break point model

Parameter	DF	Estimate	Standard Error	Likelihood Ratio 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
b_0	1	20.4040	1.6244	17.2222	23.5907	157.79	<.0001
b_1	1	0.0016	0.0002	0.0012	0.0020	55.77	<.0001
b_2	1	-0.1531	0.0177	-0.1876	-0.1183	75.02	<.0001
b_3	1	-321.644	21.6857	-364.179	-279.153	219.99	<.0001
b_4	1	-135.722	11.3590	-158.011	-113.475	142.76	<.0001
b_5	1	502.8612	35.9333	432.4946	573.3778	195.84	<.0001
b_6	1	-100.977	11.3088	-123.203	-78.8441	79.73	<.0001
b_7	1	-115.010	8.5892	-131.803	-98.1254	179.29	<.0001

TABLE 6

Comparison fit quality

Model	Deviance			BIC
	DF	Value	Value/DF	Value
Fractional polynomial	91	346.8786	3.8119	818.7929
Break - point	91	163.3524	1.7951	635.2667

7. Conclusion

The results of this study can be used also from Insurance companies and Social Security Organizations for the estimation of the total cost of the treatment of a disease.

In the first part of his paper a multistate model is proposed for health insurance in Greece. The model has three stages and the mathematical basis of the model and the basic probabilities are presented. One of the main factors for calculating actuarial values and average duration of claims is the sickness recovery intensities.

In the second part, we study sickness recovery intensities of a Greek population as a functions of two covariates, age x and sickness duration z. The data has irregular sickness recovery rates between six and eight weeks of sickness. To model these effects we propose two models, one using fractional polynomials and a second with break- point predictor terms. According the criteria we gave to evaluate the models we proposed a model with three knots for graduated the sickness recovery intensities.

In next study we will compare the Greek rates with the UK experience and we will classify the diseases in groups according sickness duration.

TABLE 7

Predicted values

Age Group	Duration (weeks)									
	1	2	3	4	5	6	7	8	9	10
20-24	7.496142	23.24207	37.80964	17.13395	12.71954	11.26617	11.60902	11.29038	11.96069	11.77625
25-29	12.19549	35.87034	55.46258	23.34875	16.29122	13.60223	14.1692	14.72589	15.96456	14.93614
30-34	10.36805	30.74306	42.16609	15.61678	11.78106	10.44923	11.19181	11.86756	13.18093	12.91313
35-39	8.743666	27.0399	42.0756	17.25441	12.64605	10.75047	11.18924	11.7773	12.44783	10.65983
40-44	7.772669	24.13772	37.41692	15.4909	11.11942	9.761757	10.62514	11.31906	11.84362	9.685329
45-49	7.856087	25.30896	41.04729	17.26567	12.99791	10.57533	11.56706	12.83538	14.77714	12.68438
50-54	7.057875	23.57335	41.01371	18.84674	14.61729	12.98597	14.20429	16.04442	17.61508	13.51257
55-59	6.54384	22.32048	41.21201	18.76501	14.49857	13.85197	15.8172	17.67587	19.29389	16.77754
60-64	2.084172	7.390384	15.6667	8.311067	6.810362	5.820637	6.587605	7.887732	8.761511	7.581646

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