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RANDOM SUMS OF BERNOULLI RANDOM VARIABLES IN MODELLING RISK FREQUENCY REDUCTION OPERATIONS

By

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Abstract

Risk frequency reduction operations play an extremely important role in treatment of risks. The main purpose of the present paper is to establish new applications of random sums of Bernulli random variables in modelling a wide class or risk frequency reduction operations of particular practical importance (JEL C51).

1. Introduction

Risk management is one of the specialties within the field of general management. As such, risk management shares many of the characteristics of general management, and yet is unique in several important respects. Given the focus on risks, risk management may be defined as the systematic process of managing an organization's risk exposures to achieve its objectives in a manner consistent with public interest, human safety, environmental factors and the law. It consists of the planning, organizing, directing and controlling activities undertaken with the intent of providing an efficient prerisk plan that minimizes the adverse impact of risk on the organization resources, earnings and cash flows, Head (1978). This broad definition makes clear that neither an organization's risk manager nor any other single executive acting alone can fully perform the entire risk management process within any organization. The cooperation of all other managers and personnel is required. For those exposures to risk which arise from an organization's production activities, for example, the cooperation of production managers in directing their subordinates to turn out a safe product in a safe manner is essential, Grose (1987). Similarly, risk exposures growing out of marketing cannot be dealt with effectively except through the efforts of marketing managers in directing those in the field to try to avoid those activities which are likely to result in risks or, once a risk has occurred, to act promptly to minimize the adverse effects from the risk, Greene and Serbein (1983).

The function of the executive charged with responsibility for risk management is not to personally minimize the adverse effects of all risks but, instead, to coordinate the efforts of all managers in reducing risks for which each of them has some responsibility and, therefore, control. Because of these relationships, it is usually said that risk management is largely a staff function, advisory and coordinative with respect to those other managers who have the line authority to control risks. In most situations, a risk manager's only line authority is within the risk management department, Gallagher (1956). A risk manager has both line and staff relationships. Within the risk management itself, the risk manager has a line relationship to subordinates, with the right to order them to perform actions relating directly to the basic purposes of the department. With respect to managers at all levels of other departments, the risk manager typically stands in a staff relationship, with the right to advice, persuade and perhaps even cajole these other managers to take actions designed to minimize the adverse effects of risks. The risk manager's right to serve in this staff capacity usually derives from top management's acknowledgment of expertise and, therefore, support. Only in extraordinary situations, such as life being immediately threatened by an extreme hazard, would a risk manager have limited line authority to, for example, halt production in order to remove the hazard.

During the last three decades, risk management expanded very rapidly on a global basis and well its traditional insurance boundaries. Risk analysis and risk assessment have have become far more sophisticated, using new tools of decision theory. Systems safety and pressures on health care institutions have created new approaches to risk and quality, for both services and products. The growing complexity of financial arrangements has led to new techniques for financing risk ranging from swaps, hedges and collars to financial re-insurance. In an increasingly complex and ever-changing world, risk management can extract from other disciplines those concepts to deal with new risks, Settembrino (1994). It is very well known that management of future uncertainty is everyone's responsibility and that the constituent disciplines or risk management include probability theory, economics, operations research, systems theory, psychology and behavioral science.

Probability theory provides risk management with very useful stochastic models. The contribution of these models to the development of risk managements as an organizational discipline has been proved very important. Random sums constitute a significant part of the stochastic models applied to risk management problems. Important research topics in the area of stochastic modelling can be considered the development, study and applications of random sums in risk analysis and risk treatment. The main purpose of the present paper is to establish applications of random sums of Bernoulli random variables in modelling some risk frequency reduction operations.

2. Random Sums of Bernoulli Random Variables

We suppose that

$\{Xn,\ n=1,2,...\}$

is a sequence of independent and identically distributed random variables, and each X_n is distributed as the Bernoulli random variable X with probability function

$$R(X=x) = \begin{Bmatrix} \alpha, & x=1 \\ 1-\alpha, & x=0' \end{Bmatrix}$$

where

$0<\alpha<1.$

It is easily demonstrated that the probability generating function of the random variable X is given by

$P_x(z) = E(z^X)$

$=\alpha z+1\text{-}\alpha,\ |z| \leq 1.$

We also suppose that N is a discrete random variable with support space given by the set of nonnegative integers and independently distributed of the sequence $\{X_n,\ n=1,2,...\}$. The random variable

$Y= X_1+X_2+...+X_N$

is called a random sum of Bernoulli random variables. If

$$P_N(z) = E(Z^N),\ |z| \leq 1 \tag{1}$$

is the probability generating function of the random variable N and

$$P_Y(z) = E\ (z^Y),\ |z| \leq 1$$

is the probability generating function of the random variable Y, then we get that

$$P_Y(z) = E(z^Y)$$

$$= E[E(z^Y|N)]$$

$$= E[(\alpha z+1-\alpha)^N]. \tag{2}$$

From (1) and (2) it follows that

$$P_Y(z) = P_N(\alpha z+1-\alpha). \tag{3}$$

Random sums of Bernoulli random variables have very interesting applications in a variety of practical disciplines and in several branches of probability theory and statistics, Feller (1971). The main purpose of the present paper is to establish new applications of random sums of Bernoulli random variables in risk treatment, which is a very important area of the fast developing discipline of risk management. More precisely, the paper makes use of the random sums in stochastic modelling of risk frequency reduction operations. Such operations attack the risk by reducing its frequency.

Empirical studies related to risk management practices of modern complex organizations, have shown that risk specialists have begun to treat risks that result from scientific research, engineering inventions, space exploration and use of computers, by making risk frequency reduction operations the most important part of their risk control programs, Kloman (1992). From such studies we can easily get the conclusion that stochastic modelling of risk frequency reduction operations can be of great importance in assessing, selecting and applying such operations.

3. Modelling of Risk Frequency Reduction

Risk identification, risk measurement and risk treatment are the steps of the risk management process. The point of risk identification activities is to locate and document the risks that are presented by the existence and operations of an organization. Risk identification requires perseverance, imagination and, above all, an awareness of changes taking place within an organization. Nearly all operational and procedural changes will in some manner create risk, reduce risk or eliminate risk. The key to effective risk identification is to be informed, Crockford (1980). After the risk manager has identified the various types of risks faced by an organization, these risks must be measured in order to determine their relative importance and to obtain information that will help the risk manager to decide upon the most desirable combination of risk treatment tools. Information is needed concerning risk frequency and risk severity. Both risk frequency and risk severity data are needed to evaluate the importance of an exposure to risk. The best way to measure risks involves probability distributions. This method makes possible comprehensive risk measurements than any other method, Bernstein (1996). Treatment of risk represents the most dynamic and challenging activity of the risk management discipline. After the risk manager has identified and measured the risks faced by an organization, he must decide how to handle them. There are two fundemental approaches. First, the risk manager can make use of risk financing tools to finance the risks that do occur. Second, the risk manager can make use of risk control tools to change the risk exposures in such a way in order to reduce an organization's expected property, liability and personal risks, or to make the annual risk experience more predictable. Risk control tools include risk avoidance, risk frequency reduction, risk severity reduction, separation, combination and some transfers. Risk frequency reduction attacks the risk by lowering its frequency. The variety of risk frequency reduction operations is illustrated by the following examples. The frequency of fire can be reduced by fire resistive construction and the frequancy of product liability suit can be reduced by tightening the quality control limits. Moreover, the frequency of an industrial accident may be reduced by safely meetings. These examples make completely clear that risk frequency reduction is fundamental in risk treatment. It is universally accepted that stochastic modelling of risk frequency reduction operations is very important in assessing, selecting and applying such operations, Head (1978). Below we establish applications of random sums of Bernoulli random variables in modelling certain risk frequency reduction operations.

In the discipline of risk management, the discrete random variable N denotes the frequency of a risk if N denotes the number of risk occurrences in a given time interval and space. We assume that a certain risk frequency reduction operation is applied to a given risk. According to this operation each risk occurrence is retained with probability α and deleted with probability 1-a, the retaining or deleting of any risk occurrence being independent of the retaining or deleting of any other risk occurrence. Under these assumptions it is readily proved that the random sum of the Bernoulli random variables

$$Y = X_1+X_2+\ldots+X_N$$

defined in the previous section, denotes the frequency of the same risk after the above risk frequency reduction operation applied.

Below we consider several particular cases of the distribution of the above random sum. These particular cases are of some practical importance in modelling risk frequency reduction operations.

We suppose that the risk frequency N follows the Poisson distribution with probability generating function

$$P_N(z) = e\lambda(z-1), \lambda>0.$$

From (3) it easily follows that the risk frequency Y, after the above risk frequency reduction operation applied, follows the Poisson distribution with probability generating function given by

$$P_Y(z) = e^{\alpha\lambda(z-1)}$$

If the random variable N follows the Poisson-uniform distribution with probability generating function

$$P_N(z) = \frac{1-e^{\lambda(z-1)}}{\lambda(1-z)},$$

then the random variable Y also follows the Poisson-uniform distribution with probability generating function given by

$$P_Y(z) = \frac{1 - e^{\alpha\lambda(z-1)}}{\alpha\lambda(1-z)}.$$

Moreover, if the random variable N follows the uniform distribution with probability generating function

$$P_N(z) = \frac{1 - z^n}{n(1-z)},$$

where

$n = 1, 2,...$

then the probability generating function of the random variable Y is given by

$$P_Y(z) = \frac{1 - (\alpha z + 1 - \alpha)^n}{\alpha n(1-z)}$$

This probability generating function belongs to a renewal distribution, Artikis (1983). More precisely, this probability generating function belongs to the renewal distribution corresponding to the distribution of the random variable S which follows the binomial distribution with probability generating function given by

$P_s(z) = (\alpha z+1-\alpha)^n$.

4. Concluding Remarks

Risk frequency reduction operations are generally recognized to be very important in risk treatment. Moreover, stochastic modelling of such operations can be of particular importance in this area of risk management. In this paper it is shown that random sums of Bernoulli random variables can be very useful models for a variety of risk frequency reduction operations. More precisely, Poisson, Poisson - uniform and some renewal random sums of Bernoulli random variables are established as being fundamental in the study and practical applications of risk frequency reduction operations.

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