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Real Convergence using TAR Panel Unit Root Tests: An Application to The Southern African Development Community

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Abstract

The African Union advocates the creation of a monetary union in stages for the entire continent starting with each of the different sub-regions. Building from that, the Southern African Development Community has, among its goals, the creation of a monetary union for its member states. However, the recent European Union crisis has sparked renewed interest in the achievement of convergence among potential member states prior to the establishment of a monetary union. We examine real convergence in per capita output of the Southern African Development Community countries using annual data from 1980 to 2017. An approach that combines threshold modelling, panel data unit root testing and critical values bootstrapping is used. The sample is divided in three sets comprising the high, middle and low-income countries. We find significant convergence among the high-income countries. However, after adding successively the middle and low-income countries to the set of high-income states, we find no evidence of convergence. These results support that the member states of this community do not conform to the optimal currency area criteria, casting doubt on the establishment of an efficient monetary union in the short run.

Keywords: real convergence, panel data unit root test, bootstrap, threshold model, SADC.
JEL Classification: C12, C33, F43

1. Introduction

One of the goals of the African Union (AU) is to create a monetary union in stages for the entire continent starting with each of the different sub-regions (Masson and Patillo, 2005). Building from that, one of the ultimate goals of the Southern African Development Community (SADC)[1] is the establishment of a monetary union. The community has agreed on a set of macroeconomic criteria in order to monitor progress towards convergence (Bala, 2011).

According to Solow (1956) and subsequent literature, economic integration, under free factor mobility and international diffusion of technological knowledge, will automatically promote economic convergence. However, a different view is that integration will increase regional

[1] SADC comprises Angola, Botswana, Democratic Republic of Congo, Malawi, Namibia, Lesotho, Madagascar, Mauritius, Mozambique, Seychelles, Swaziland, South Africa, Tanzania, Zambia and Zimbabwe.

and geographical disparities as the factors of production will be concentrated in the more developed region as a result of increasing returns to scale and externalities (Romer, 1986; Romer, 1990; Krugman, 1990).

A fast and automatic economic convergence allows free market forces to erode regional inequalities. This renders unnecessary the transfer of funds from the richer member states to the poorer ones of the monetary union. However, if this convergence does not take place or is not sufficiently rapid, there is a need of an explicit regional policy in favour of the less developed economies of the union as long as this allows a reduction of inequalities (Beyaert, 2003).

Since the seminal papers of Mundell (1961), McKinnon (1963), Kennen (1969) and Ingram (1973); there is a sizable literature that analyses the conformity of countries to the optimum currency area (OCA) criteria. For the SADC region, we can note the contribution, among others, of Jefferis (2007), Kumo (2011), Zerihun et al. (2012) and Zerihun et al. (2014). Looking at convergence, the concept involved traditionally an analysis of whether the real per capita incomes of poor countries are catching up with those of the rich countries. Since recent decades, there was a shift from that traditional view as regional economic integration required the strengthening of macroeconomic policies (Kumo, 2011). De Haan et al. (2008) emphasize the importance of symmetric business cycles among countries forming a monetary union. This is crucial as considerable divergence will imply a non-optimal common monetary policy for all the member states. Furthermore, the recent Greek crisis, which threatened the whole Euro zone, showed the importance of a proper assessment of potential candidates for a monetary union.

The aim of this article is to assess convergence in SADC real per capita output using annual data from 1980 to 2017[2]. A reason for analysing co-movements in real output is that countries facing high correlations of cyclical movements of real output do not need country specific monetary and exchange rate policies (Tavlas, 2009) and can therefore be successful in forming a currency union.

The article uses a non-linear bootstrap extension of the Evans and Karras (1996a) approach. As explained by Beyaert and Camacho (2008), there is a belief that the convergence process may not be uniform as countries may only converge once certain institutional, economic and political conditions are put in place. Another possibility is that convergence may take place at a specific rate under certain conditions and at another rate under other conditions; justifying the use of a non-linear technique.

The convergence hypothesis is conducted following the strategy used by Beyaert and Camacho (2008). We start with a set of high-income countries for which convergence is highly probable. Once confirmed, we add countries to this set and repeat the convergence test on this augmented group. We find that the threshold autoregressive (TAR) specification outperforms the linear specification while testing convergence among the SADC rich countries. The TAR results support significant absolute convergence with some short-lived periods of conditional convergence prior the nineties. Next, we add the set of middle and low-income countries to the first set. We find that the linear specification performs better than its non-linear counterpart. However, there is no significant convergence found. The SADC as a whole is characterised by significant divergence, casting doubt on the

[2] Convergence test can also be conducted using exchange rates. However, this article focuses solely on real output per capita.

establishment of an efficient monetary union in the short run. Therefore, the SADC countries do not conform to the OCA criteria.

The rest of the paper is organized as follows. Section 2 looks at a brief literature review focused on convergence in Africa with a particular focus on the SADC region. Section 3 explains the Evans and Karras' (1996a) linear framework. Section 4 describes the TAR extension of the Evans and Karras' (1996a) method. Section 5 uses both methods to test for convergence in the per capita output of SADC countries from 1980 to 2017. Section 6 concludes.

2. A Brief Literature Review

There are two concepts of convergence in the literature. The first concept explains convergence as a catching up process in gross national product (GNP) per capita in order to achieve an alignment of the standards of living in the different participant states of a monetary union (Wagner, 2013). Barro and Sala-i-Martin (2004) have demonstrated that countries having lower starting values of capital-labor ratio have higher per capita growth rates and tend to catch up with those having higher capital-labor ratios. Thus, countries are said to converge if the poor countries with lower initial income grow faster than others (Kumo, 2011; Vakulenko, 2016). This is the beta (β) convergence concept.

The second concept looks at cross-sectional dispersion. Convergence will therefore occur if the dispersion, proxied for example by the standard deviation of the logarithm of per capita income across a group of countries, declines over time. This is the sigma (σ) convergence concept. Although β-convergence tends to generate σ-convergence, Barro and Sala-i-Martin (2004) have shown that β-convergence is a necessary but not a sufficient condition for σ-convergence.

There will be absolute convergence whenever per capita incomes of countries converge to a unique steady state value, implying an equalization of incomes between the countries. Conditional convergence, on the contrary, implies different steady states to which per capita incomes of countries converge. According to Varblane and Vahter (2005), absolute convergence will occur in countries with similar initial levels of income and similar economic, political and social structures; leading to σ-convergence or club-convergence.

The conventional approach of testing convergence estimates, for a sample of countries, cross-sectional relationship between the growth rate of output per capita and the initial level of output over some period; with the possibility of controlling for other variables (Evans and Karras, 1996b). Evans (1995) shows that this approach produces valid inference under unrealistic conditions: the dynamic structures of the different countries should have identical first order autoregressive representations; every economy affects every other economy completely symmetrically; and the vector of variables controls for all permanent cross-economy differences. Evans and Karras (1996a) and Evans (1998) propose the use of a panel data approach.

Using unit root and cointegration tests for panel data, McCoskey (2002) investigates the convergence properties of six well-being indicators in Sub-Saharan Africa namely government share of GDP, capital per worker, openness of the economy, real GDP per capita, standard of living and real GDP per worker from 1960 to 1990. The countries were divided into clubs and the author has also investigated convergence among SACU and SADC countries. Little evidence of long-run relationship was found for income based variables, except for real GDP per capita and for some sub-clubs such as the one comprising South Africa and Malawi. Using unit root tests, no evidence of convergence was found for real

GDP based variables, nor for government share of GDP and real GDP based variables for SACU and SADC countries.

Kumo (2011) has used the concept of β and σ convergence to analyse real GDP absolute and conditional convergence in SADC countries for the period 1992-2009. No convergence was found for the SADC as a whole, nor for the CMA countries comprising South Africa, Lesotho, Namibia and Swaziland. However, while considering individual countries, convergence towards common stochastic trends was found for South Africa and Botswana.

Zerihun et al. (2012) have used the triple test in order to analyse each SADC member states' business cycles for symmetry and evaluate the countries' ratio of relative intensity of co-movements in business cycles with co-SADC countries and versus that of major trading partners. They have found that, as not all SADC countries conform to OCA criteria, a common monetary policy will not be optimal for the community as a whole, especially in the short-run.

3. Review of the linear framework[3]

3.1 The basic Evans-Karras procedure

Evans and Karras (1996a) test real convergence in a panel data using the following specification:

$$\Delta g_{n,t} = \delta_n + \rho_n g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i} \Delta g_{n,t-i} + \varepsilon_{n,t}, \quad (1)$$

with *n=1,…, N* and *t=1,…, T;* where *n* refers to cross-sections and *t* to time periods. The variable $g_{n,t}$ is defined as:

$$g_{n,t} = y_{n,t} - \bar{y}_t; \quad (2)$$

where $y_{n,t}$ is the per capita income of country *n* in logs and $\bar{y}_t$ is the cross-country average.

A $\rho_n = 0$ implies that the *N* countries diverge, whereas $0 < -\rho_n < 1$ for all *n* is a convergence condition. Beyaert (2005) has shown that divergence of one single country in the panel implies that the $g_{n,t}$ will be $I(1)$ for all *n*. The convergence will be absolute if $\delta_n = 0$ for all *n* whereas it will be conditional if not.

Evans and Karras (1996a) obtain an estimate of the standard deviation (s_n) and use it to transform the data to $w_{n,t} = g_{n,t}/s_n$. They obtain the estimate of ρ and its t-ratio by applying OLS to:

$$\Delta w_{n,t} = \delta_n + \rho w_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i} \Delta w_{n,t-i} + \varepsilon_{n,t} \quad (3)$$

If this t-ratio is sufficiently negative, reject the null in the test:

$$\rho_n = 0 \ \forall n \ \ against \ \rho_n < 0 \ \forall n \quad (4)$$

Under the alternative, the economies converge. Otherwise, they diverge. This is similar to a test of panel unit root. If divergence is rejected in (3), test the null that:

$$\delta_n = 0 \ \forall n \ \ against \ \delta_n \neq 0 \ \forall n; \quad (5)$$

[3]An adaptation of Beyaert and Camacho (2008). Refer to the article for full description.

for some n in equation (1). For that purpose, estimate this equation for $n=1,...,N$; compute $\Phi=\frac{1}{N-1}\sum_{n=1}^{N}(t_{\hat{\delta}_n}^2)$; and reject the null if Φ is too large, then convergence will be conditional. Otherwise, convergence is absolute.

If the errors in (1) are contemporaneously uncorrelated, the tests for (4) and (5) will have asymptotic distributions whenever N and T tend to infinity. Evans and Karras (1996a) suggest an improvement that use critical values derived through simulations from Normal independent distributions.

Beyaert and Camacho (2008) identify two limitations of the above approach. First, cross-sectional dependence will not hold with a lower N. Second, the assumption of linearity is unrealistic as some countries may have experienced profound changes during the period under study. To address these limitations, they have considered two simulations extensions. The first relaxes the assumption of cross-sectional independence building on Chang (2004) results. The second considers a nonlinear specification.

3.2 A bootstrap version of Evans-Karras approach

Beyaert and Camacho[4] (2008) models (1) using a Seemingly Unrelated Regression Estimation (SURE) form as:

$$\Delta G = X\beta + \varepsilon, \tag{6}$$

where $X = \left(\breve{\iota}, \breve{G}_{-1,} \Delta\breve{G}_{-1,} \dots, \Delta\breve{G}_{-p}\right)$, with components:

$$\breve{\iota} = \begin{bmatrix} \bar{1}_T & & 0 \\ & \ddots & \\ 0 & & \bar{1}_T \end{bmatrix}, \text{ with } \bar{1}_T = [1, \dots, 1]'_{Tx1};$$

$$\breve{G}_{-1} = \begin{bmatrix} G_{1,-1} & & 0 \\ & \ddots & \\ 0 & & G_{N,-1} \end{bmatrix}, \text{ where } G_{n,-1} \text{ is } G_n \text{ lagged one period; similarly}$$

$$\Delta\breve{G}_{-i} = \begin{bmatrix} \Delta G_{1,-i} & & 0 \\ & \ddots & \\ 0 & & \Delta G_{N,-i} \end{bmatrix}, \text{ for } i=1, \dots, p \text{ where } \Delta G_{n,-i} \text{ is } \Delta G_n \text{ lagged } i \text{ periods.}$$

An estimate $\widehat{\Omega} = [s_{nm}]$ can be computed, with $s_{nm} = \frac{1}{T}\sum_{t=1}^{T} e_{nt}e_{mt}$ for $n, m=1,..., N$; where e_{nt} is the OLS residual of model (6) corresponding to observation t for country n. The Feasible Generalised Least Squares (FGLS) estimator of β is then:

$$\hat{\beta}_{FGLS} = \left[X'\hat{V}^{-1}X\right]^{-1} X'\hat{V}^{-1}\Delta G; \tag{7}$$

where $\hat{V} = \widehat{\Omega} \otimes I_T$.

The hypothesis of divergence against convergence is tested by estimating model (6) under the restriction that $\rho_n = \rho$ for all n. The p-value is obtained by bootstrap. For that purpose, an FGLS estimate of model (1) is obtained under the additional restriction that $\rho = 0$, with the

[4] We are grateful to these authors for making the Gauss codes available.

residuals recentered and arranged in a matrix[5], then resampled with replacement to obtain a new time series of residuals for each n that preserves the initial contemporaneous correlation (Maddala and Wu, 1999; Chang, 2004). Bootstrap data are obtained using the FGLS estimates of model (1) under $\rho = 0$. The value of the test statistic is computed in each replication in the same way as on the observed data.

The bootstrapped version of test (5) is carried out in a similar way and the following test-statistic is computed as:

$$\Phi=\frac{1}{\text{N-1}}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_{FGLS,n})\right]^2\right\}, \tag{8}$$

where $\hat{\delta}_{FGLS,n}$ is the FGLS estimate of δ_n in (6). Then, (6) is estimated under the restriction that $\delta_n = 0$ for all *n*, and the residuals are recentered and resampled by row. The bootstrap data are generated from these bootstrap residuals under this restriction and the corresponding bootstrap Φ statistics and p-values are computed.

4. Convergence analysis with TAR models[6]

4.1 The non-linear model

It could be that the *N* countries converge only if certain institutional, political or economic conditions are fulfilled. It may happen that $0 < -\rho_n < 1$ for all n under certain circumstances but that $\rho_n = 0$ if these circumstances are not met. Convergence can also take place at different rates. It may happen that $0 < -\rho_n < 1$ for all *n* but that its specific value differs according to the prevailing conditions at time *t*. This can be specified as follows:

$$\Delta g_{n,t} = [\delta_n^I + \rho_n^I g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i}^I \Delta g_{n,t-i}] I_{\{z_{t-1}<\lambda\}}$$

$$+ \left[\delta_n^{II} + \rho_n^{II} g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i}^{II} \Delta g_{n,t-i}\right] I_{\{z_{t-1}\geq\lambda\}} + \varepsilon_{n,t}, \tag{9}$$

with *n =1*, …, *N*; and *t =1*, …, *T*. In this model, $I\{x\}$ is an indicator which takes the value of 1 when *x* is true and zero otherwise. It acts as a dummy variable which takes a unit value if the condition $z_{t-1} < \lambda$ is fulfilled. So when $z_{t-1} < \lambda$, the model is $\Delta g_{n,t} = \delta_n^I + \rho_n^I g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i}^I \Delta g_{n,t-i} + \varepsilon_{n,t}$, whereas it is $\Delta g_{n,t} = \delta_n^{II} + \rho_n^{II} g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i}^{II} \Delta g_{n,t-i} + \varepsilon_{n,t}$ when $z_{t-1} \geq \lambda$. So at any *t*, the dynamics of the per capita incomes follow one of the two possible regimes. I will call "regime *I*" the case where $z_{t-1} < \lambda$ and "regime *II*" the case where $z_{t-1} \geq \lambda$. The parameter λ is a "threshold" parameter and equation (9) belongs to the class of TAR models first introduced by Tong (1978). Model (9) includes the linear model (1) as a particular case, which takes place when z_{t-1} stands on the same side of λ for all *t*. The threshold parameter is usually unknown. In order to carry out the estimation process, the restriction that $0 < \pi_1 \leq P(z_{t-1} \leq \lambda) \leq 1 - \pi_1$ is imposed so that no regime takes place in less than a π_1 fraction of the total sample, with π_1 typically around 0.10 or 0.15. If π_1 falls below this limit, the linear process is preferred.

Beyaert and Camacho (2008) propose two extensions. The first uses a multivariate panel data model instead of a single one. The second refers to the possible non-stationarity of the data in

[5]For each country *n*, the sample mean over time is subtracted from the residuals to obtain zero-mean residuals.
[6]An adaptation of Beyaert and Camacho (2008). Refer to the article for full description.

the form of a unit root in the individual series when $\rho_n = 0$. The second extension has been considered by Caner and Hansen (2001) too although their model is limited to a single series.

Divergence will occur in model (9) if $\rho_n^I = \rho_n^{II} = 0$ for all n. Alternatively, global convergence would correspond to $0 < -\rho_n^i < 1$ for all *n* and *i*=*I*, *II*. Finally, there would be partial convergence if $0 < -\rho_n^i < 1$ but $\rho_n^j = 0$ for all *n* and *i*≠*j*.

In (9), the so-called transition variable, z_t, can be either endogenous when its values are directly obtained from the $g_{n,t}$ variables; or exogenous. In the endogeneity case, it makes sense to choose $z_t = g_{m,t} - g_{m,t-d}$, for some m country and some $0 < d \leq p$. Note that the *m* country and *d* are not a priori fixed but rather determined endogenously. In this way, from the statistical point of view, z_t would be stationary, whether the economies converge, $g_{n,t} \sim I(0)$ for all n and both regimes; or not, $g_{n,t} \sim I(1)$, for one or both regimes. Economically, it means that the shift from one regime to another is related to the growth rate of country *j* in the last *d* periods. The equation (9) also assumes that all the parameters change when the economies shift from regime *I* to regime *II*. However, restricted versions of this specification could be considered such as:

$$\Delta g_{n,t} = \delta_n\left[\rho_n^I g_{n,t-1}\right] I_{\{z_{t-1}<\lambda\}} + \left[\rho_n^{II} g_{n,t-1}\right] I_{\{z_{t-1}\geq\lambda\}} + \sum_{i=1}^{p} \varphi_{n,i} \Delta g_{n,t-i} + \varepsilon_{n,t}; \quad (10)$$

assuming that only the convergence rate varies with the regime.

In (9) and (10), p is assumed to be high enough so that $\varepsilon_{n,t}$ is a white noise process for each *n*; excluding serial correlation. However, cross-country contemporaneous correlation cannot be excluded.

4.2 Estimation

Model (9) is estimated using concentration in a GLS approach given the dependence of the coefficients on the threshold value of the transition variable and the structures of the variance-covariance matrix of the residuals.

Suppose that *λ*, *m* and *d* are known and their values are collected in vector $\theta_0 = (\lambda_0, m_0, d_0)'$. So, conditional on θ_0, model (9) can be seen as an equation with known dummy variables. If the total number of available time observations for each country is (*T*+*p*+1) so that (*p*+1) initial values prior to *t*=1 exist and ⊙ denotes an element-by-element multiplication; (9) can be written in SURE form as:

$$\Delta G = \left[X \odot \breve{I}_{I,\theta_0} \;\vdots\; X \odot \breve{I}_{II,\theta_0}\right] \begin{bmatrix} \beta_{I,\theta_0} \\ \cdots \\ \beta_{II,\theta_0} \end{bmatrix} + \varepsilon; \quad (11)$$

where β_{I,θ_0} (β_{II,θ_0}) is defined as a vector of parameter β in which the parameters of each country are stacked by type in a column. They refer to the coefficients under regime *I* (*II*) when $\lambda = \lambda_0$, $m = m_0$ and $d = d_0$. The expression $\breve{I}_{I,\theta_0}$ refers to an (*NT* x 1) vector obtained by stacking *N* times the (*T* x 1) dummy variable vector:

$$I_{I,\theta_0} = [I_{z_{0,p<\lambda_0}}, I_{z_{0,p+1<\lambda_0}}, \cdots, I_{z_{0,T-1<\lambda_0}}]'; \quad (12)$$

where $z_{0,t} = g_{m_{0,t}} - g_{m_{0,t-d0}}$. Similarly, $\breve{I}_{II,\theta_0}$is obtained by stacking *N* times the vector

$$I_{II,\theta_0} = [1 - I_{z_{0,p<\lambda_0}}, 1 - I_{z_{0,p+1<\lambda_0}}, \cdots, 1 - I_{z_{0,T-1<\lambda_0}}]'. \quad (13)$$

Model (11) can be written more compactly as:

$$\Delta G = \breve{X}_{\theta_0} \beta_{\theta_0} + \varepsilon \quad (14)$$

Estimating this model by FGLS is justified by the characteristics of the variance-covariance matrix of the residuals. So:

$$\beta_{\theta_{0,FGLS}} = \left[\breve{X}'_{\theta_0}\hat{V}_0^{-1}\breve{X}_{\theta_0}\right]^{-1}\breve{X}'_{\theta_0}\hat{V}_0^{-1}\Delta G; \tag{15}$$

where $\hat{V}_0 = \hat{\Omega}_0 \otimes I_T$ and $\hat{\Omega}_0 = [s_{nm,0}]$ with $s_{nm,0} = \frac{1}{T}\sum_{t=1}^{T} e_{nt,0}e_{mt,0}$ for *n*, *m*=1, …, *N*; and with $e_{it,0}$ being the OLS residuals of model (15) corresponding to observation *t* for country *n*.

In practice, the true values of λ, m and d are unknown. However, appropriate values can be inferred from the data. Let us posit $\hat{e}_{\theta_0}$ the FGLS residuals vector of model (11) and define the weighted sum of squared residuals $S^2_{\theta_0} = \frac{1}{T}\varepsilon'_{\theta_0}\hat{V}_0\hat{\varepsilon}_{\theta_0}$. Since this is a function of θ_0, a grid search procedure can be applied to obtain:

$$\hat{\theta} \equiv [\hat{\lambda}, \hat{m}, \hat{d}] = \arg\ \min_{\theta_0}(s^2_{\theta_0});$$

and the least squares estimates of the other parameters can be obtained by plugging in the point estimate $\hat{\theta}$ in model (11) and obtain the corresponding $\hat{\beta}_{\hat{\theta}_{0,FGLS}}$.

The grid search procedure is implemented for each $m \in [1, 2, \dots, N]$ and each $d \in [1, 2, \dots, p]$ by giving to λ the value $(g_{m,\tau} - g_{m,\tau-d})$ for each $\tau \in [1, 2, \dots, T]$. The fraction of the sample falling in the implied regime *I* is then computed. If this fraction lies in the interval $[\pi_1, 1 - \pi_1]$, the corresponding FGLS estimator of β_0 and the weighted sum of residuals are computed. If not, this combination of m, d and λ is discarded and the procedure goes to the next point of the grid. Once all the points have been checked, the estimation process ends, obtaining $\hat{\theta}$ and the corresponding $\hat{\beta}_{\theta_{0,FGLS}}$. This is the "grid-FGLS" method. Once model (9) is estimated, its superiority to the linear Evans-Karras method model (1) has to be checked. If confirmed, the next task will be to test if there is convergence or not by applying some type of unit-root test on the ρ coefficients of (9). If there is evidence of convergence, the last step should test absolute against conditional convergence through a test on the δ coefficients of (9)[7].

4.3 Convergence tests

If model (9) is empirically favoured, the next step consists of testing convergence against divergence. The null hypothesis of the test is:

$$H_{0,2} : \rho_n^I = \rho_n^{II} = 0 \ \forall n \tag{16}$$

If not rejected, it reflects that the countries diverge both under regime *I* and regime *II*. There are three types of alternatives of economic interest that can be tested:

$$H_{A,2a} : \rho_n^I < 0, \rho_n^{II} < 0 \ \forall n, \tag{17a}$$

$$H_{A,2b} : \rho_n^I < 0, \rho_n^{II} = 0 \ \forall n, \tag{17b}$$

$$H_{A,2c} : \rho_n^I = 0, \rho_n^{II} < 0 \ \forall n, \tag{17c}$$

The alternative (17a) reflects convergence of the countries both under regime *I* and *II*. This is the case of "full convergence". The alternatives (17b) and (17c) imply that convergence takes place only under regime *I* or only under regime *II* respectively. This is the case of "partial convergence".

[7] The linearity test procedure can be found in Beyaert and Camacho (2008).

Note that both the null and the alternative hypothesis are assuming that the coefficients satisfy the same property for all the countries at a time. This is consistent with the definition (2) of the series $g_{n,t}$. Since these series are in deviations from their common cross-section mean, as soon as one of the country does not converge to the other, even though the remaining countries do converge to each other; none of the $g_{n,t}$ series can be *I*(0). In other words, the $g_{n,t}$ series of the panel are all *I*(0) or all *I*(1).

To discriminate between the three alternatives, several statistics are used along the lines of Caner and Hansen (2001) for the single-equation case. They propose a Wald-type statistics for the test against the global alternative $H_{A2_a}$of convergence. Extending their proposition to the panel data case, the statistic is:

$$R_2 = t_I^2 + t_{II}^2, \tag{18}$$

where t_I and t_{II} are t-type statistics associated with the estimation of ρ_n^I and ρ_n^{II}, respectively, in model (9). If $\hat{\rho}_n^i$ is the grid-GLS estimate of ρ_n^i for each regime *i*, then:

$$t_i = \frac{\hat{\rho}_n^i}{s_{\rho_n^i}}, \tag{19}$$

for *i*=*I*, *II*. Given the definition of R_2, large values of this statistic are favourable to convergence. For the alternative of partial convergence $H_{A,2b}$, the statistic to be used would be t_I while t_{II} would be used to test against the partial convergence hypothesis $H_{A,2c}$. These are left-sided tests. So, if $t_I(t_{II})$ is too small, whereas $t_{II}(t_I)$ is not, the data favour the hypothesis of convergence under regime *I* (*II*) and divergence under regime *II* (*I*). Bootstrap simulations are used in order to find the appropriate p-values. The statistics R_2, t_I and t_{II} are computed and sorted in ascending order to obtain the bootstrap p-values. The last step of the convergence analysis consists of discriminating between absolute and conditional convergence. The absolute convergence hypothesis refers to the fact that converging countries share the same steady path. Conditional convergence refers to the existence of parallel, though not coincident, paths. So, in terms of model (9), under the maintained hypothesis that $\rho_n^i < 0$, ∀ *n=1,...,N* and *i = I, II;* absolute convergence is equivalent to:

$$\delta_n^i = 0,\ \forall n = 1, \dots, N, i = I, II \tag{20}$$

If the convergence process is partial, in the sense that it takes place only under one of the two regimes, say regime *I*, then absolute convergence would correspond to:

$$\delta_n^I = 0,\ \forall n = 1, \dots, N \tag{21}$$

Note, however, that in this two regimes model, another case of interest occurs when $\rho_n^i < 0$ for all *n* and *i* (global convergence) but $\delta_n^i = 0$ for only one value of *i*. In this case, convergence is absolute under one regime although conditional under the other one. Several tests statistics are used to discriminate between these different cases. The proposed tests are based on the grid-GLS estimation of model (9). They are direct extensions to the TAR model of the statistics proposed by Evans and Karras (1996a) for the linear case and are derived from the t-statistics, $t(\hat{\delta}_n^i) = \frac{\hat{\delta}_n^i}{s_{\hat{\delta}_n^i}}$, with *i* = *I, II* and *n=1, ..., N*; associated with the estimated value of the constant terms. They are the following:

$$\Phi_a = \frac{1}{2N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_n^I)\right]^2 + \sum_{n=1}^{N}\left[t(\hat{\delta}_n^{II})\right]^2\right\} \tag{22a}$$

$$\Phi_b = \frac{1}{N-1}\left\{\sum_{n=1}^{N}\left[t(\hat{\delta}_n^I)\right]^2\right\} \tag{22b}$$

$$\Phi_c = \frac{1}{N-1}\left\{\sum_{n=1}^{N}\left[t\left(\hat{\delta}_n^{II}\right)\right]^2\right\} \quad (22c)$$

Given the endogeneity of the transition variable, here too the bootstrap p-values are obtained from adjusting a linear model to the observed data. A null constant term is imposed in model (1) and the following specification is estimated:

$$\Delta g_{n,t} = \rho_n g_{n,t-1} + \sum_{i=1}^{p} \varphi_{n,i}\Delta g_{n,t-i} + \varepsilon_t\ ; \quad (23)$$

with *n=1*, …, *N* and *t=1*, …, *T* by feasible GLS. The matrix of recentered residuals is then resampled by row and the bootstrapped data are generated from the estimates of (26). Model (9) is adjusted on these data and the three tests Φ_a, Φ_b and Φ_c are computed. The bootstrap right-sided p-values are extracted from their empirical distributions. The three statistics are then used in the following way:

- If $H_{0,2}$ has been rejected in favour of $H_{A,2a}$:
  - Φ_a is too large: conditional convergence takes place under both regimes.
  - Φ_b is too large, although Φ_c is not: conditional convergence takes place under regime *I* and absolute convergence takes place under regime *II*.
  - Symmetrically $\Phi_c$is too large, although $\Phi_b$is not: conditional convergence takes place under regime *II* and absolute convergence takes place under regime *I*.
- If $H_{0,2}$ has been rejected in favour of $H_{A,2b}$:
  - Φ_b is too large: conditional convergence takes place under regime *I.*
  - Φ_b is not large enough: absolute convergence takes place under regime *I.*
- If $H_{0,2}$ has been rejected in favour of $H_{A,2c}$:
  - Φ_c is too large: conditional convergence takes place under regime *II.*
  - Φ_c is not large enough: absolute convergence takes place under regime *II.*

5. Empirical Results

5.1 Data

The article uses annual data of logarithms of real per capita GDP of fifteen SADC countries from 1980 to 2017. The data are from the World Bank development indicators and are expressed in constant 2010 US dollars. The countries are divided into three sets. The first set, called the rich or high-income countries, comprises Botswana, Seychelles, Mauritius, South Africa and Namibia. The second set, called the middle-income countries, comprises Zambia, Swaziland, Angola and Lesotho. The last set, called the poor or low-income countries, comprises Zimbabwe, Tanzania, Mozambique, Madagascar, Malawi and the Democratic Republic of Congo (D. R. Congo). The countries were grouped under these sets given their average output per capita.

5.2 Results

To test for convergence, we follow the strategy used by Beyaert and Camacho (2008). Indeed, working with the deviation of per capita output from a common cross-country means that the divergence of one country implies that the whole set of $g_{n,t}$ series are *I*(1). We start with the first set of high-income countries for which convergence is highly probable. Once confirmed, we add countries to this set and repeat the convergence test on this augmented group. We carry on with this process until the last set comprises all SADC countries. The data of the five richer countries are presented in figure 1. The real GDP per capita of South Africa and Namibia were falling at first. However, since the mid-nineties, there is a tendency of convergence between these countries. Indeed, the gap between their outputs per capita becomes smaller over time. It is therefore reasonable to expect convergence for this first set of countries following the different tests.

Figure 1: Output evolution of the five richer countries (in logs)

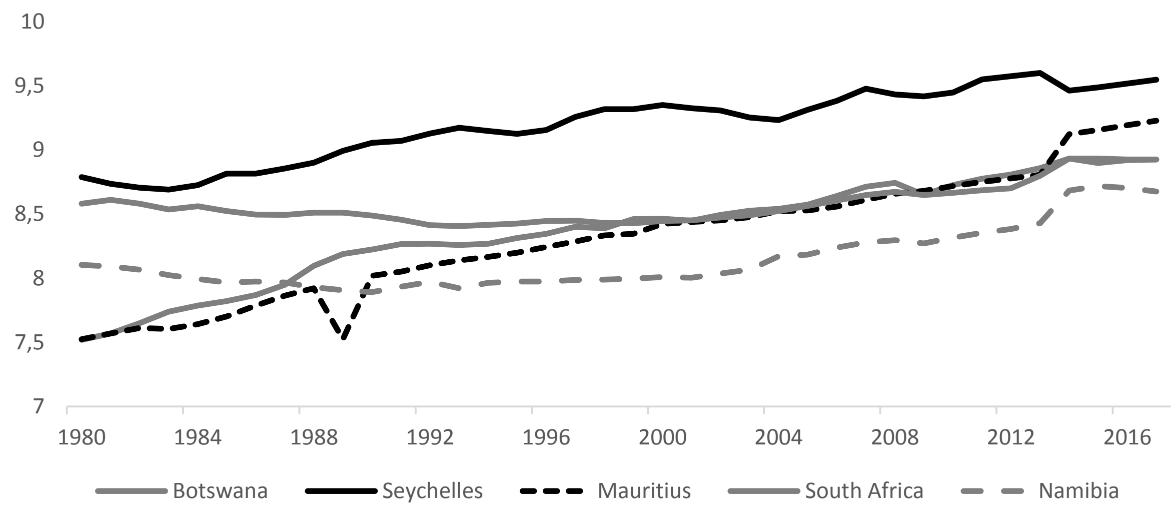


Table 1 presents the results of the convergence tests for the first set of countries. The statistical results are gathered in table 1.1 for the linear model (1) and in table 1.2 for the TAR model (9). The linear model shows strong convergence between countries as the null hypothesis of divergence is strongly rejected. The convergence is absolute during the whole period under study as the statistic is insignificant based on the bootstrapped p-values. We next consider the non-linear test. The test of linearity rejects strongly the null hypothesis and favours the TAR model for both unrestricted and restricted specifications. Thus, the process is better described by model (9). Botswana is the country whose evolution dictates the switch from one regime to the other. From figure 1, it can be seen that Botswana started at a lower position, together with Mauritius. The situation has since changed and Botswana ended up being among the top three richer countries of the SADC. So, Botswana is a good representative of the intensity of the convergence process; justifying its choice to form the transition variable. The estimate of the delay parameter d is 1 so that the transition variable is $g_{Botswana,t} - g_{Botswana,t-1}$. The threshold parameter λ is estimated at 4.33. This implies that regime *I*, which takes place in 82.86% of the sample, corresponds to the years in which the growth rate of Botswana's output per capita was 4.33 percentage points below the average growth rate of the other countries of the set. By the same token, regime *II*, which takes place in 12.2% of the sample; corresponds to the years in which Botswana's growth rate was 4.33 percentage points higher. There is convergence under both regimes as the hypothesis of divergence is strongly rejected, leading to a full convergence. However, the convergence under regime *I* seems to be stronger than convergence under regime *II*. The absolute versus conditional convergence test indicates that convergence was absolute under regime *I* and conditional under regime *II*.

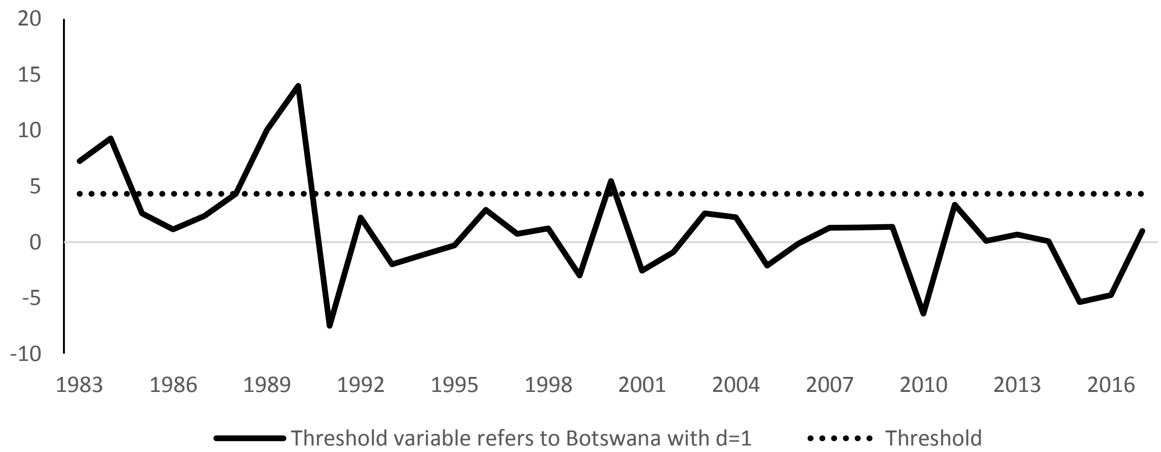
Figure 2 identifies the periods that correspond to each regime, the threshold position and the value of the transition variable for the high-income countries. Regime *I* dominated between 1985-1988 and after 1989, except a short-lived deviation in 2000. Regime *II* took place during the early and late 80's. These countries have converged towards a common steady state path during the last 25 years while, prior to this, they each converged towards their individual steady states. This makes sense as prior 1985, Botswana and Mauritius output per capita were the lowest of the set. The weighted average of the output per capita of these high-income countries is economically meaningful and can be used as a benchmark in the analysis of convergence towards this. This is the next step.

Table 1: Output results of the five richer countries

1.1 Linear Model					
Divergence vs Convergence			Absolute vs Conditional Convergence		
0.000			0.269		
Convergence			Absolute		
1.2 TAR Model					
Linearity Test					
Unrestricted			Restricted		
0.000			0.000		
Transition Country		λ		% Observation in Regime I	
Botswana		4.33		82.86	
Convergence Tests					
Divergence vs Convergence			Absolute vs Conditional Convergence		
Regime I	Regime II	Both	Regime I	Regime II	Both
0.000	0.003	0.005	0.424	0.057	0.104
Full Convergence			Regime I Absolute and Regime II Conditional		

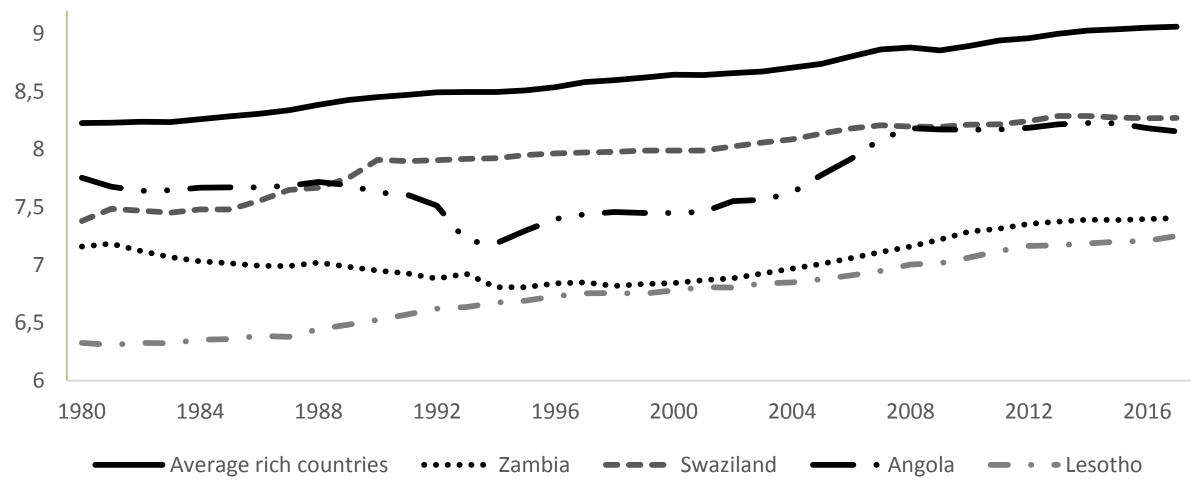
Note: Entries refer to bootstrap p-values. The selected lag length is 1 has chosen by the Ljung-Box statistic. Delay parameter (d) = 1.

Figure 2: Threshold of the five richer countries



We analyse the convergence of the middle-income countries towards the average output per capita of the first set of rich countries. We present their output per capita in figure 3. Angola has witnessed a large fall of output prior 1995 due to years of civil war. Zambia has also experienced a fall of real output prior the late nineties. However, all these countries have managed to slightly narrow the gap with the average of the high-income countries despite a slight divergence from 2014.

Figure 3: Output evolution of the average of the five richer and the middle-income countries



Tables 2.1 and 2.2 present the statistical results for this second set of countries. The linear results do not support convergence between these countries. Indeed, the test does not reject the null hypothesis of divergence. Therefore, there is no need to test for absolute or conditional convergence. Table 2.2 focuses on the TAR results. The test of linearity does not reject the null hypothesis. As the linear model outperforms the TAR, there is no need to present the results of the latter. Therefore, the output of the middle-income countries do not converge towards the average output of the high-income states.

Table 2: Results of the five rich and middle-income countries

2.1 Linear Model					
Divergence vs Convergence			Absolute vs Conditional Convergence		
0.576			-		
Divergence					
2.2 TAR Model					
Linearity Test					
Unrestricted			Restricted		
0.953			0.908		
Transition Country		λ		% Observation in Regime I	
-		-		-	
Convergence Tests					
Divergence vs Convergence			Absolute vs Conditional Convergence		
Regime I	Regime II	Both	Regime I	Regime II	Both
-	-	-	-	-	-

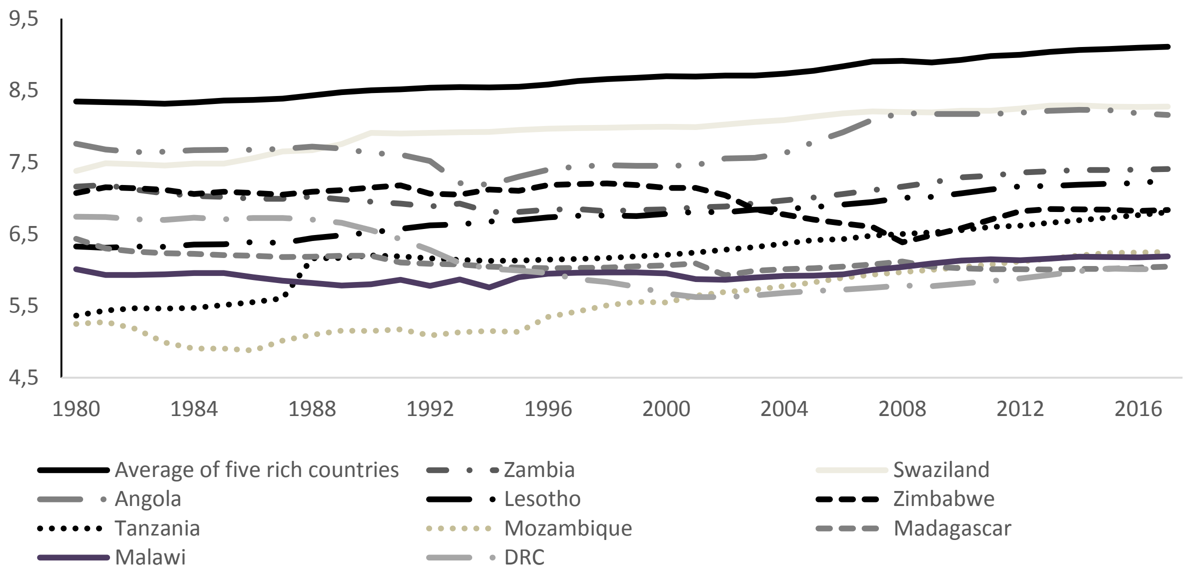
Note: Entries refer to bootstrap p-values. The selected lag length is 1 has chosen by the Ljung-Box statistic. Delay parameter (d) = 1.

We extend the previous analysis with the inclusion of the third set of low-income countries. Figure 4 presents the evolution of output per capita of these countries and the average of the rich. Zimbabwe started well with a narrow gap. However, from 1991, the country witnessed a drop in output per capita which reached its lowest in 2008. There was a recovery following that year but growth remained sluggish after 2013. The D. R. Congo witnessed a meagre recovery from 2001 following years of poor performance that started in the late eighties.

Despite that, the gap widened as the output per capita remained constant from 2015 onwards[8]. Mozambique and Tanzania have managed to narrow the gap as the two countries experienced some steady growth since the mid-eighties for the former and the mid-nineties for the latter. However, the gap remains large. In general, there is a weak tendency of narrowing the gap these past years for most of the countries. This is exacerbated by the increase of the high-income countries average output. This may jeopardize the probability of convergence. We present next the results for this last set of countries comprising the entire SADC community.

The results for the SADC community as a whole are gathered under table 3. The results for the linear model are presented under table 3.1. The hypothesis of divergence is not rejected given the bootstrapped p-values. There is therefore no need to present further results of this linear test. The TAR results are presented under table 3.2. The linearity test indicates that the linear model outperforms the TAR specification. Therefore, the middle and low-income SADC countries do not converge to the output per capita of the five high-income countries based on the linear results. It is important to note that that the null hypothesis of divergence is rejected while using standard p-values for the second set that comprises the middle-income countries only and for the last set that comprises the entire SADC community. These results indicate the importance of addressing issues of serial correlation and cross-sectional dependence in a panel setting to avoid spurious results. As these countries are from the same geographical region, they may be subject to common shocks[9].

Figure 4: Output evolution of the average five rich countries and other SADC countries



[8]Convergence tests were conducted without Zimbabwe and without Zimbabwe and D. R. Congo. Although the linear model outperformed the TAR model, no convergence was found for the remaining countries. The results are available upon request.

[9] The standard p-values are 0.004 for the set of middle-income countries and 0.000 for the entire community, supporting a strong convergence.

Table 3: Results of all the SADC countries

3.1 Linear Model					
Divergence vs Convergence			Absolute vs Conditional Convergence		
0.321 Divergence			-		
3.2 TAR Model					
Linearity Test					
Unrestricted			Restricted		
1.000			1.000		
Transition Country		λ	% Observation in Regime I		
-		-	-		
Convergence Tests					
Divergence vs Convergence			Absolute vs Conditional Convergence		
Regime I	Regime II	Both	Regime I	Regime II	Both
-	-	-	-	-	-

Note: Entries refer to bootstrap p-values. The selected lag length is 1 has chosen by the Ljung-Box statistic. Delay parameter (d) = 1.

6. Conclusion

This article uses the Beyaert and Camacho (2008) non-linear extension of the Evans and Karras (1996a) approach to test for real convergence in SADC annual real output per capita from 1980 to 2017. The methodology uses a threshold model, panel data unit root tests and bootstrap critical values in order to test for convergence. This allows the possibility of obtaining non-linear convergence and correcting both serial correlation and cross-sectional dependence.

The test is first conducted on a set of five high-income countries for which convergence is highly probable. This set comprises Botswana, Seychelles, Mauritius, South Africa and Namibia. The results show that the TAR model outperforms the linear specification. These countries are characterized by significant absolute convergence during the past 25 years.

Next, the first set is updated by adding a second list of middle-income countries of the region. This new set comprises Zambia, Swaziland, Angola and Lesotho. We analyze convergence of real output per capita of these countries towards the average output per capita of the high-income members. Although the linear model outperforms the TAR specification, we find no convergence towards the average output of the high-income countries.

A last set of countries comprising Zimbabwe, Tanzania, Mozambique, Madagascar, Malawi and the Democratic Republic of Congo; is added to the previous one. This set represents the SADC community as a whole. Again, the linear model outperforms the TAR specification. However, no convergence was found.

Given these findings, the SADC community, as a whole, does not conform to the OCA criteria. The member states have therefore different characteristics, making convergence difficult. The SADC community needs to reinforce and monitor the progress of member states towards the achievement of macroeconomic targets as agreed. Only when convergence is achieved then the establishment of a monetary union can follow with far less risks of destabilisation through exogenous shocks.

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