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Global Economic Growth and Environmental Change

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Abstract

The purpose of this paper is to examine dynamic interdependence between wealth accumulation, capital accumulation, economic structure, domestic and global division of labor, international trade and environmental change with transboundary pollution. It analyzes not only inequalities in income, wealth and economic structures between (any number of) countries, but also differences in environmental changes between countries. The model is built on the basis of the Solow model, the Uzawa two-sector growth model, the Oniki-Uzawa trade model, and the neoclassical growth models in environmental economics. The general equilibrium dynamic model includes any number of national economies and each national economy has two sectors. After building the model, we show that the dynamics of the world economy with J countries is described by $2J$ differential equations. We simulate the motion of the global economy with three national economies. We also examine the effects of changes in the propensity to environmental taxes, the efficiency of environmental protection, and the population upon dynamic paths of the system. Our analysis provides some insights into the complexity of global economic growth with environment. For instance, the well-known study by Grossman and Krueger (1995) identifies no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that this conclusion holds for some countries, but is invalid for other countries.

JEL Classification: F11; O41; Q56.
Keywords: trade; economic growth; transboundary pollution; environmental change; environmental policies.

1. Introduction

The purpose of this study is to examine dynamic interdependence between economic growth, economic structural changes, inequalities in income and wealth, international trade, and environmental change over time. Issues related to growth, environment and international relations have recently increasingly caught attention from economists as well as the public. For instance, in association with rapid trades between Japan and China, the environmental pollution of Japan by China has increasingly become a hot public issue in Japan. Global warming is not only a single country's concern. Roles of governments on complexity of dynamic interactions between growth, trade, and environment are also changing rapidly in the world. As economic and environmental systems interact with complicated relations, it is necessary to deal with the economic system as integrated whole rather than separated subsystems. The purpose of this study is to develop a multi-country growth model with endogenous physical capital and wealth accumulation and environmental change. The study analyzes not only inequalities in income, wealth and economic structures among (any number of) countries, but also differences in environmental changes among countries. The economic system is built on the basis of the Solow model, the Uzawa two-sector model, and the Oniki-Uzawa trade model, and the neoclassical growth model in environmental economics. Different from the growth models with the Ramsey approach, we use an alternative utility function proposed by Zhang (1993) to determine saving and consumption.

There are different economic factors that may affect environmental change (e.g., Levinson and Taylor, 2008; Lamla, 2009; Gassebner *et al*. 2011). Production and consumption deteriorate environmental quality and bad environment lowers productivities and utilities. There are many studies on dynamic interdependence among economic growth, consumption and environmental changes (John and Pecchenino, 1994; and Prieur, 2009). In economics dynamic relations between growth and environmental change have been formally analyzed since the publication of the seminal papers by Keeler et al. (1971), Ploude (1972) and Forster (1973). Although the neoclassical framework of optimal economic growth with pollution has been generalized in different ways (e.g., Gruver, 1976; Becker, 1982; Masu, 1987; van der Ploeg and Withagen, 1991; Tahvonen and Kuuluvainen, 1993; Selden and Song, 1995; Bovenberg and de Mooij., 1997; Schou, 2002; Chen et al. 2009), almost all of these studies don't take account of possible effects of trade on pollution and growth. In this study, we consider dynamic interactions among environment, growth and international trade. Environment is determined not only by behavior of firms, households, transboundary pollution and the government, but also affects behavior of firms, households, trade patterns and the government. Although economic growth worsens environmental conditions, growth also implies a higher material standard of living which will, through the demand for a better environment induces changes in the structure of the economy to improve environment. As pointed out by Lin and Liscow (2012: 268): "The effects of increasing income on environmental quality is an issue that has long puzzled economists. For over decade, economists have theorized that a graph of environmental degradation versus income often looks something approximating an inverted-U shape, dubbed the environmental Kuznets curve (EKC) after Simon Kuznets' work in the 1950s and 1960s on income equality (Kuznets, 1966, 1965)." In recent years environmental issues have received more attention (e.g., Lin and Liscow, 2012). Tsurumi and Managi (2010) observe that there are three effects that are significant for examining dynamics of environmental pollution and resource use. These effects are: (i) increases in output tends to require more inputs and produce more emissions; (ii) changes in income or preferences may lead to policy changes which will affect production and thus emission; and (iii) as income increases, the economic structure may be changed which will causes changes in the environment. It is argued that the net effect of these effects tends to result in the environmental Kuznets curve, even though a large number of empirical studies find different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, a monotonically increasing or monotonically decreasing relationship - between pollution and rising per capita income levels (e.g., Bravo and Marelli, 2007). Another important aspect that has been often neglected in the literature of formal growth models with environment is related to international trade and transboundary pollution. This study examines interactions among environment, productivities and trade patterns. Poor and rich countries may be different in their concerns about environmental degradation. As observed by Fairbrother (2013: 910), "Recent survey research argues that richer people are greener – that residents of more economically developed countries, as well as relatively wealthier people within countries, are more concerned about the state of the natural environment and more willing to pay to protect." Nevertheless, the research by Fairbrother (2013: 910) concludes that "environmental concern is generally higher in poorer countries, and there is no relationship over time between economic development and people's willingness to pay for environmental protection. Within countries, richer people are slightly more concerned about the environment, but only on some dimensions and not others." There are actually different views on relationships among economic growth, values, inequality and environment (e.g., Inglehart, 1995; Brechin, 1999; Gelissen, 2007; Henderson and Millimet, 2007; Dunlap and York, 2008; Franzen and Meyer, 2010; Givens and Jorgenson, 2011). Theoretical economics fails to address these issues with a comprehensive framework with microeconomic foundation. As pointed out by Fullerton and Kim (2008), existing research has proposed a number of different models for analyzing different questions not in an integrated way. This study deals with growth, trade, and environmental change within an integrated framework.

In regard to capital mobility and trade, our model is based on the neoclassical growth trade model. According to Findlay (1984), almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time. It is well known that the classical Ricardian theory of comparative advantage and the Heckscher-Ohlin theory are not dynamic since labor and capital stocks (or land) are exogenous. Early trade models with capital movements are originated by MacDougall (1960) and Kemp (1961). But those models in the 1960s are mostly static. Moreover, most of trade models with endogenous capital and/or knowledge in the contemporary literature are either limited to two-country or small open economies (for instance, Grossman and Helpman, 1991; Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998). It is necessary to deal with trades and growth with an analytical framework with any number of countries as the world does consist of many countries and trades occur among multiple countries. There are some neoclassical growth models with international trade. For instance, Oniki and Uzawa (1965) and Bardhan (1965) examine trade patterns between two economies in a Heckscher-Ohlin model with fixed savings rates. Deardorff and Hanson (1978) propose a two country trade mode with different saving rates across countries. There are some other growth models with international trade (e.g., Brecher et al., 2002 ; Nishimura and Shimomura, 2002 ; Bond *et al.* 2003; Ono and Shibata, 2005). None of these models with endogenous capital accumulation contains endogenous environmental changes.

This study is primarily concerned with dynamic interdependence between wealth and physical capital accumulation, environmental change, and trade patterns between countries. Each national economy is basically described by the Uzawa two-sector growth model. As far as capital accumulation and trade pattern determination are concerned, our study is influenced by the Oniki-Uzawa framework, even though we extend the Oniki-Uzawa model to include endogenous environmental change. We also deviate from the traditional approach in modeling behavior of households. This study applies an alternative approach to consumer behavior by Zhang (1993). The model in this study is a further development of the two models by Zhang. Zhang (2012) proposed a multi-country model with capital accumulation. Although Zhang (2013) introduced environment into the growth theory proposed by Zhang, the model was limited to a national economy. This study synthesizes the main ideas in these two models. This paper is organized as follows. Section 2 defines the multi-country model with physical capital and environmental change. Section 3 shows that the world with J economies is described by $2J$ differential equations and also simulates the model. Section 4 carries out comparative dynamics analysis in regard to some parameters. Section 5 concludes the study.

2. The multi-country trade growth model with environmental change

The world economy consists of multiple countries, indexed by $j = 1, ..., J$. Country j has a fixed population, N_j. Each country has three sectors: one capital goods sector, one consumer goods sector and one environmental sector. Like in Zhang (2013), the national government financially supports the environmental sector. The capital goods and consumer goods sectors are the same as in the Uzawa two sector model (Uzawa, 1961). In describing the production sectors, we follow the neoclassical growth theory (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). It is assumed that all the countries produce homogenous capital goods. This study extends the Oniki-Uzawa trade model with the Uzawa two-sector model. As reviewed by Ikeda and Ono (1992), most of trade models with endogenous capital is structured like Oniki-Uzawa trade model and its various extensions with one capital goods. Each country also has one consumer goods (and service) sector. The output of this sector is not tradable in the international markets. Households own assets of the economy and distribute their incomes to consume and save. Production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors pay environmental taxes and sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. We require savings and investment to be equal at any point of time. Let prices be measured in terms of the capital goods and the price of the capital goods be unit. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$. Capital goods are be used as inputs in the three sectors. Capital depreciates at a constant exponential rate δ_j, being independent of the manner of use within each country. Let $p_j(t)$ denote the price of consumer goods. We use subscript index, i, s and e to stand for capital goods sector, consumer goods sector, and environmental sector, respectively. We use $N_{jm}(t)$ and $K_{jm}(t)$ to stand for the labor force and capital stocks employed by sector m in country j. Let $F_{jm}(t)$ stand for the output level of sector m in country j.

2.1 The capital goods sectors

We assume that the production of capital goods is to combine labor force and physical capital with constant technology. We use the conventional production function to describe a relationship between inputs and output, except that environment affects productivity. The production function is specified as follows

$$F_{ji}(t) = A_{ji} \Gamma_{ji}(E_j) K_{ji}^{\alpha_{ji}}(t) N_{ji}^{\beta_{ji}}(t), \; A_{ji}, \alpha_{ji}, \beta_{ji} > 0, \; \alpha_{ji} + \beta_{ji} = 1, \tag{1}$$

where A_{ji}, α_{ji}, and β_{ji} are positive parameters. Here, $\Gamma_{ji}(E_j)$ is a function of the environmental quality measured by the level of pollution, $E_j(t)$, in country j. It is reasonable to assume that the productivity of the capital goods sector is non-positively related to the pollution level, i.e., $d\Gamma_{ji} / dE_j \le 0$. It should be remarked that this type of production functions with pollution as an augment is well used in the literature of growth with endogenous pollution (e.g., Adu, 2013).

We use τ_{ji} to stand for the fixed tax rate on the capital goods sector. The marginal conditions of the capital goods sector are given by

$$r(t) + \delta_k = \frac{\alpha_{ji} \bar{\tau}_{ji} F_{ji}(t)}{K_{ji}(t)}, \; w_j(t) = \frac{\beta_{ji} \bar{\tau}_{ji} F_{ji}(t)}{N_{ji}(t)}, \tag{2}$$

where $\bar{\tau}_{ji} \equiv 1 - \tau_{ji}$, $0 < \tau_{ji} < 1$.

2.2 The consumer goods sectors

The production function of the consumer goods sector is

$$F_{js}(t) = A_{js} \Gamma_{js}(E_j(t)) K_{js}^{\alpha_{js}}(t) N_{js}^{\beta_{js}}(t), \; \alpha_{js} + \beta_{js} = 1, \; \alpha_{js}, \beta_{js} > 0, \tag{3}$$

where A_{js}, α_{js}, and β_{js} are the technological parameters of the consumer goods sector and $\Gamma_{ji}(E_j(t))$ is a function of the environmental quality. We use τ_{js} to stand for the fixed tax rate on the consumer goods sector and introduce $\bar{\tau}_{js} \equiv 1 - \tau_{js}$, $0 < \tau_{js} < 1$.

The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_{js} \bar{\tau}_{js} p_j(t) F_{js}(t)}{K_{js}(t)}, \; w_j(t) = \frac{\beta_{js} \bar{\tau}_{js} p_j(t) F_{js}(t)}{N_{js}(t)}. \tag{4}$$

2.3 Environmental change

We measure a country's environmental level by its stock of pollutants, $E_j(t)$. We specify the dynamics of the stock of pollutants as follows

$$\dot{E}_j(t) = \theta_{ji} F_{ji}(t) + \theta_{js} F_{js}(t) + \theta_j C_j(t) - F_{je}(t) - \bar{\theta}_j E_j(t) + \Omega_j((E_q(t))), \tag{5}$$

in which θ_{ji}, θ_{jx}, θ_j, and $\bar{\theta}_j$ are positive parameters and

$$F_{je}(t) = A_{je}\,\Gamma_{je}(E_j(t))K_{je}^{\tilde{\alpha}_{je}}(t)N_e^{\tilde{\beta}_{je}}(t), \;\; A_{je}, \tilde{\alpha}_{je}, \tilde{\beta}_{je} > 0, \tag{6}$$

where A_{je}, α_{je}, and β_{je} are positive parameters, and $\Gamma_{je}(E_j)$ (≥ 0) is a function of E_j. The term $\theta_{ji} F_{ji}$ means that pollutants that are emitted during production processes are linearly positively proportional to the output level (for instance, Gutiérrez, 2008). The parameter, θ_{ji}, means that in consuming one unit of the good the quantity θ_{ji} is left as waste. Hence, $\theta_{ji} F_{ji}$ is the creation of pollutants by the capital goods sector. Similarly, $\theta_{js} F_{js}$ is the creation of pollutants by the consumer goods sector. The creation of pollutants by consumers is given $\theta_j C_j$. The parameter $\bar{\theta}_j$ is called the rate of natural purification. The term $\bar{\theta}_j E_j$ measures the rate that the nature purifies environment. The term, $K_{je}^{\tilde{\alpha}_e} N_{je}^{\tilde{\beta}_e}$, in F_{je} means that the purification rate of environment is positively related to capital and labor inputs. The function, Γ_{je}, implies that the purification efficiency is dependent on the stock of pollutants. It is not easy to generally specify how the purification efficiency is related to the scale of pollutants. For simplicity, we require Γ_e to be positively related to the stock of pollutants. In an economy where environment is heavily polluted, the environment sector is productive is the sense that some efforts may bring about great results. On the other hand, efforts to improve environment may have little impact on the clean environment.

As far as economic production, capital accumulation and environmental dynamics for a national economy are concerned, our model is similar to the dynamic model by Dinda (2005) in many aspects. Like in Dinda's model, we allow capital allocation between commodity production and pollution abatement; but different from Dinda's model which omits labor and neglects possible pollution due to consumption, we allow labor allocation between commodity production and pollution abatement and explicitly treat consumption as a source of pollution. It is important to take the pollution due to consumption into consideration when dealing with relations between environment and growth. Another important difference between our international trade model and Dinda's national growth model is that our model takes account of transboundary pollution. In the literature of trade and environment there are many studies which explicitly take account of transboundary pollution (e.g., Copeland and Taylor, 1995, 2003; Ono, 1998; Di Maria and Smulders, 2004; Takarada, 2005; Schweinberger and Woodland, 2008; Qiu and Yu, 2009; Abe, *et al.* 2012). We measure the effects of transboundary pollution by $\Omega_j((E_q))$ where is a function of pollutants of all the countries. The functions are possibly related to many factors, such as distances between countries and wind directions. We will specify these functions when simulating the model.

2.4 Consumer behaviors

This study models household behavior with the approach proposed by Zhang (1993). In addition to the environmental taxation on firms (outputs), we also take account of taxation on wealth income, consumption and wage income. There are models with environmental tax incidence (see, for instance, Rapanos, 1992, 1995). We will show that our approach differs from the traditional approaches also in regard to how the environmental taxation affects behavior of households. Consumers make decisions on choice of consumption levels of goods as well as on how much to save. Let $\bar{k}_j(t)$ stand for per capita wealth of country j. We have $\bar{k}_j(t) = \bar{K}_j(t)/N_j$, where $\bar{K}_j(t)$ is the total wealth owned by country j. We use τ_{jk} and τ_{jw} to respectively stand for the tax rates on the wealth income and wage income. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, and the wage payment $w_j(t)$, is

$$y_j(t) = \bar{\tau}_{jk}\, r(t)\bar{k}_j(t) + \bar{\tau}_{jw}\, w_j(t),$$

where $\bar{\tau}_{jk} \equiv 1 - \tau_{jk}$ and $\bar{\tau}_{jw} \equiv 1 - \tau_{jw}$. The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{7}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. The disposable income is used for saving and consumption. At each point of time, a consumer would distribute the total available budget between saving $s_j(t)$ and consumption $c_j(t)$. The budget constraint is given by

$$(1 + \tau_{jc})p_j(t)c_j(t) + s_j(t) = \hat{y}_j(t), \tag{8}$$

where τ_{jc} is the tax rate on consumption. In the literature of environmental economics there are different taxes on households as well as producers (e.g., Bovenberg and Smulders, 1995; Bovenberg *et al*., 2008; and Heutel, 2012).

The consumers choose two variables, $s_j(t)$ and $c_j(t)$, to maximize utilities. We specify the utility function as follows

$$U_j(t) = \Gamma_j(E_j(t))c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}, \quad \xi_{0j}, \lambda_{0j} > 0, \tag{9}$$

where $\Gamma_j(E_j(t))$ is a function related to the environment, ξ_{0j} is the propensity to consume and λ_{0j} the propensity to own wealth. As in Balcao (2001) and Nakada (2004), we assume that utility negatively depends on pollution. Different from the models by Balcao (2001) and Nakada (2004) in which pollution is a side product of the production process, in our model environmental change is affected by the production, consumption, transboundary pollution, natural purification and government environmental policies. Here, we neglect the possibility that consumers make decisions with consideration of improving environment. For instance, consumers may prefer environment-friendly goods to other goods. With regard to how much money the economic agent should spend on environmental improvement, Selden and Song (1995) hold that at a lower level of pollution, the representative agent does not care much about environment and spends his resource on consumption; however, as the environment becomes worse and income becomes higher, more capital will be used for environmental improvement. As reviewed by Munro (2009: 43), "environmental economics has been slow to incorporate the full nature of the household into its analytical structures. … [A]n accurate understanding household behavior is vital for environmental economics." Our modelling framework makes a progress toward this direction.

The household maximizes $U_j(t)$ subject to the budget constraint (8) yields

$$p_j(t)c_j(t) = \xi_j \, \hat{y}_j(t), \;\; s_j(t) = \lambda_j \, \hat{y}_j(t), \tag{10}$$

where

$$\xi_j \equiv \frac{\rho_j \, \xi_{0j}}{1 + \tau_{jc}}, \;\; \lambda_j \equiv \rho_j \, \lambda_{0j}, \;\; \rho_j \equiv \frac{1}{\xi_{0j} + \lambda_{0j}}.$$

We now find dynamics of capital accumulation. According to the definition of $s_j(t)$ the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{11}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

2.5 The capital and labor employed by the environment sector

We now determine how the government determines the number of labor force and the level of capital employed for improving environment. We assume that all the tax incomes are spent by the environmental sector. The government's tax incomes consist of the tax incomes on the production sector, consumption, wage income and wealth income. The government's income is

$$Y_{je}(t) = \tau_{ji} \, F_{ji}(t) + \tau_{js} \, F_{js}(t) + \tau_{jc} \, c_j(t)N_j + \tau_{jw} \, w_j(t)N_j + \tau_{jk} \, r(t)\bar{k}_j(t)N_j \, . \tag{12}$$

For simplicity, we assume that the government's income is used up only for the environmental purpose. As there are only two input factors in the environmental sector, the government budget is given by

$$\left(r(t) + \delta_k\right)K_{je}(t) + w_j(t)N_{je}(t) = Y_{je}(t). \tag{13}$$

We need an economic mechanism to analyze how the government distributes the tax income. We assume that the government will employ the labor force and capital stocks for purifying environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is given by

$$\underset{\{K_{je}(t),\, N_{je}(t)\}}{\mathrm{Max}} F_e(t) \quad \text{s.t.:}$$

The optimal solution is given by

$$\left(r(t) + \delta_k\right)K_{je}(t) = \alpha_{je} \, Y_{je}(t), \;\; w_j(t)N_{je}(t) = \beta_{je} \, Y_{je}(t), \tag{14}$$

where

$$\alpha_{je} \equiv \frac{\tilde{\alpha}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}, \quad \beta_{je} \equiv \frac{\tilde{\beta}_{je}}{\tilde{\alpha}_{je} + \tilde{\beta}_{je}}.$$

2.6 Demand and supply

The demand and supply equilibrium for the consumer goods sector is

$$c_j(t)N_j = F_{js}(t), \quad j = 1, ..., J. \tag{15}$$

We use $K(t)$ to stand for the capital stocks of the world economy. The total capital stock employed by country j, $K_j(t)$, is allocated between the three sectors. It should be noted that $K_j(t)$ may not equal $\bar{K}_j(t)$ which is the wealth owned by country j. As full employment of labor and capital is assumed, we have

$$K_{ji}(t) + K_{js}(t) + K_{je}(t) = K_j(t), \quad N_{ji}(t) + N_{js}(t) + N_{je}(t) = N_j. \tag{16}$$

The total capital stocks employed by the world is equal to the wealth owned by the world. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t)N_j. \tag{17}$$

The world production is equal to the world net savings. That is

$$S(t) - K(t) + \sum_{j=1}^{J} \delta_{jk} K_j(t) = F(t), \tag{18}$$

where $S(t) \equiv \sum_{j=1}^{J} s_j(t)N_j$, $F(t) \equiv \sum_{j=1}^{J} F_j(t)$.

The trade balances of the economies are given by

$$B_j(t) = (\bar{K}_j(t) - K_j(t))r(t). \tag{19}$$

When $B_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $B_j(t)$ is zero, country $j's$ trade is in balance.

We completed the model. Irrespective of the obvious strict assumptions in our model, from a structural point of view the model is quite general in the sense that some well-known models in economics can be considered as its special cases. For instance, if the population is homogeneous and environment is constant, our model is structurally similar to the neoclassical growth models by Solow (1956) and Uzawa (1961). Our model is also structurally similar to the Oniki-Uzawa trade model (Oniki and Uzawa, 1965). As mentioned before, our approach is also based on some growth models in the literature of environmental economics.

3. The world economic dynamics

The dynamic system consists of any (finite) number of economies. The dynamic system is of high dimension. The following lemma shows that the dimension of the dynamical system is twice the number of countries. The reason is that each country has two accumulated variables, wealth and environmental stock. We also provide a computational procedure for calculating all the variables at any point of time. Before stating the lemma, we introduce new variables $z_j(t)$ by

$$z_j(t) \equiv \frac{r(t) + \delta_k}{w_j(t)}, \quad j = 1, ..., J.$$

Lemma 1

The dynamics of the world economy is governed by the following $2J$ dimensional differential equations system with $z_1(t)$, $\{\bar{k}_j(t)\}$, and $(E_j(t))$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \cdots, H_J(t))$, as the variables

$$\dot{z}_1(t) = \Phi_1(z_1(t), (E_j(t)), \{\bar{k}_j(t)\}),$$
$$\dot{\bar{k}}_j(t) = \Phi_j(z_1(t), (E_j(t)), \{\bar{k}_j(t)\}), \quad j = 2, ..., J,$$
$$\dot{E}_j(t) = \Omega_j(z_1(t), (E_j(t)), \{\bar{k}_j(t)\}), \quad j = 2, ..., J, \tag{20}$$

in which Φ_j and Ω_j are unique functions of $z_1(t)$, $\{\bar{k}_j(t)\}$, and $(E_j(t))$, defined in Appendix. For any given positive values of $z_1(t)$, $\{\bar{k}_j(t)\}$, and $(E_j(t))$, at any point in time, the other variables are uniquely determined by the following procedure: $r(t)$ and $w_j(t)$ by (A2) $\rightarrow$ $p_j(t)$ by (A4) $\rightarrow$ $\bar{k}_1(t)$ by (A19) $\rightarrow$ $K_j(t)$ by (A17) $\rightarrow$ $N_{ji}(t)$ and $N_{je}(t)$ by (A11) $\rightarrow$ $N_{js}(t)$ by (A7) $\rightarrow$ $K_{je}(t)$, $K_{js}(t)$, and $K_{ji}(t)$ by (A1) $\rightarrow$ $\hat{y}_j(t)$ by (A5) $\rightarrow F_{ji}(t)$, $F_{js}(t)$ and $F_{je}(t)$ by the definitions $\rightarrow$ $c_j(t)$ and $s_j(t)$ by (10) $\rightarrow$ $Y_{je}(t) = w_j(t)N_{je}(t)/\beta_{je} \rightarrow K(t) = \sum_j \bar{k}_j(t)N_j \rightarrow \bar{K}_j(t) = \bar{k}_j(t)N_j$ $\rightarrow B_j(t) = (\bar{K}_j(t) - K_j(t))r(t) \rightarrow U_j(t)$ by the definitions.

The lemma provides a computational procedure for plotting the motion of the economic system with any number of countries. We have the dynamic equations for the world economy with any number of countries. The system is nonlinear and is of high dimension. It is difficult to generally analyze behavior of the system. We simulate the model to illustrate properties of the dynamic system. We consider the world economy consists of three national economies. We specify the functions dependent on environmental quality as follows

$$\Gamma_{jm}(E_j(t)) = E_j^{-b_{jm}}(t), \; \Gamma_j(E_j(t)) = E_j^{-b_j}(t), \; j = 1, 2, 3, \; m = i, s, e.$$

We require $b_{ji}, b_{js}, b_j \geq 0$ and $b_{je} \leq 0$. The transboundary pollution functions are specified as

$$\Omega_q\left(\left(E_q(t)\right)\right) = \sum_{j,\, j\neq q}^{J} \theta_{jq}\, E_j(t).$$

It is reasonable to require $\theta_{jq} \geq 0$. The transboundary pollution functions imply that a country may be polluted by other countries and the speed is linearly related to the pollutant levels of these countries. We specify the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 30 \end{pmatrix},\ \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.7 \\ 1 \\ 0.8 \end{pmatrix},\ \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 0.9 \\ 0.7 \end{pmatrix},\ \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.9 \end{pmatrix},\ \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 0.31 \\ 0.31 \\ 0.31 \end{pmatrix},\ \begin{pmatrix} \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix},$$

$$\begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.55 \\ 0.5 \end{pmatrix},\ \begin{pmatrix} b_{1i} \\ b_{2i} \\ b_{3i} \end{pmatrix} = \begin{pmatrix} b_{1s} \\ b_{2s} \\ b_{3s} \end{pmatrix} = \begin{pmatrix} b_{1e} \\ b_{2e} \\ b_{3e} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.02 \\ 0.01 \end{pmatrix},\ \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ 0.12 \\ 0.11 \end{pmatrix},\ \begin{pmatrix} \tau_{1k} \\ \tau_{2k} \\ \tau_{3k} \end{pmatrix} = \begin{pmatrix} \tau_{1w} \\ \tau_{2k} \\ \tau_{3k} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.02 \\ 0.02 \end{pmatrix},$$

$$\begin{pmatrix} \xi_{02} \\ \tau_{jc} \\ \tau_{ji} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.01 \\ 0.01 \end{pmatrix},\ \begin{pmatrix} \tau_{js} \\ \tilde{\alpha}_{je} \\ \tilde{\beta}_{je} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.4 \\ 0.2 \end{pmatrix},\ \begin{pmatrix} b_j \\ \theta_{ji} \\ \theta_j \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.08 \\ 0.03 \end{pmatrix},\ \begin{pmatrix} \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.04 \end{pmatrix},$$

$$\theta_{jq} = 0.01,\ \ j \neq q,\ \ j, q = 1, 2, 3. \tag{21}$$

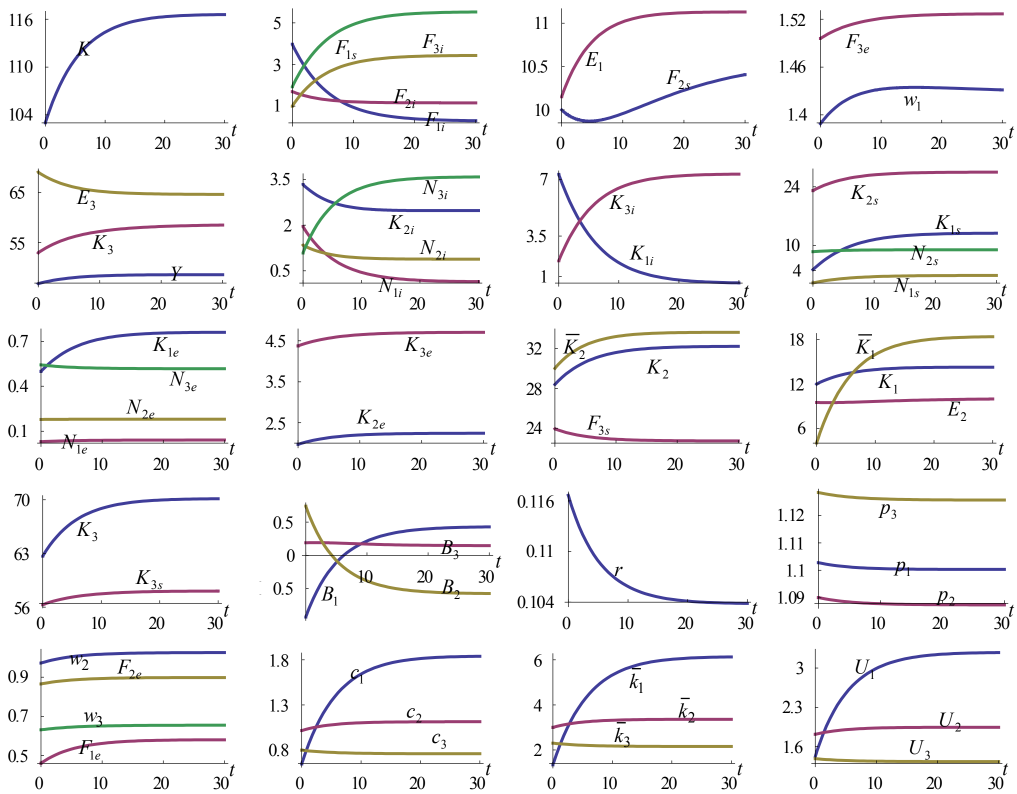
Country $1, 2$ and $3's$ populations are respectively $3, 10,$ and 30. Country 3 has the largest population. Country $1, 2$ and $3's$ total productivities of the capital goods sectors, A_{ji}, are respectively $1.7, 1,$ and 0.8. Country $1, 2$ and $3's$ total productivities of the consumer goods sectors, A_{js}, are respectively $1.5, 0.9,$ and 0.7. Country $1, 2$ and $3's$ total productivities of the environmental sector, A_{je}, are respectively $1.2, 1,$ and 0.9. Country 1 has highest total productivity; country 2 next and country 3 lowest. We call the three countries respectively as developed, industrializing, and underdeveloped economies (DE, IE, UE). We specify the values of the parameters, α_{ji}, in the Cobb-Douglas productions approximately 0.3. The DE's propensity to save is highest; country 2 next and country 3 lowest. We require the tax rates on consumption level of any country to be one percent. The tax rates on other sectors and wealth are one or two percent. We plot the motion of the system under (17) with the following initial conditions

$$z_1(0) = 0.12,\ \bar{k}_2(0) = 3,\ \bar{k}_3(0) = 2.3,\ E_1(0) = 10,\ E_2(0) = 9.5,\ E_3(0) = 53.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the global output is

$$Y(t) = \sum_j \{F_{ji}(t) + p_j(t) F_{js}(t)\}.$$

Figure 1. The Motion of the Economic System



The global output and capital stocks are increased with the given initial conditions over time, even though different sectors in different countries experience different paths of economic development due to capital accumulation and international trade. The rate of interest falls in association with rises in the global capital. The prices of services in the three economies fall. The DE's and IE's utility levels are increased, while the UE's utility level is slightly reduced. The change rends in the utility levels are in the same directions as the changes in the wealth and consumption levels of the corresponding economies. Hence, different countries experience different paths of environmental changes for the given initial conditions. The DE's trade balance is improved, while the other two economies' trade balances are deteriorated. The UE's environment is improved, while the other two countries' environment deteriorates over time. This result also implies the necessity of introducing multiple countries into the dynamic analysis of environmental change. For instance, in a study by Grossman and Krueger (1995), they pay attention to the relationship per capita and various environmental indicators - urban air pollution, the state of the oxygen regime, in river basins, and fecal contamination of river basins, and contamination of river basins by heavy metals. Grossman and Krueger (1995: 353) claim that they find no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for some national economies, but is invalid for some other economies even when the world experiences global growth. It should be noted that much of the discussion of income convergence in the literature of economic growth and development is based on the insights from analyzing models of closed economies (Barro and Sala-i-Martin, 1995). It is obviously strange to discuss issues related to global income and wealth convergence with a framework without international interactions. The reason for this is that there are few growth models with endogenous wealth and trade on the basis of microeconomic foundation. For instance, the Solow model of closed economies predicts that convergence in income levels among closed countries is achieved by faster accumulation of physical capital in the poor countries. Nevertheless, if poor countries are opened to trade, the convergence may be stopped. As shown in Figure 1, different countries will not experience convergence in per capita income, consumption and wealth in the long term as they are different in preferences and total productivities.

We also simulate the model with other initial conditions. We notice that all the variables tend to become stationary over time with different initial conditions. This hints on that the system has a stable equilibrium point. We identify the following equilibrium point

$$K = 116.6, \; Y = 48.64, \; r = 0.104,$$

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} 10.59 \\ 10.05 \\ 58.66 \end{pmatrix}, \; \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0.43 \\ 0.15 \\ -0.58 \end{pmatrix}, \; \begin{pmatrix} Y_{1e} \\ Y_{2e} \\ Y_{3e} \end{pmatrix} = \begin{pmatrix} 0.18 \\ 0.48 \\ 1.02 \end{pmatrix}, \; \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.10 \\ 1.09 \\ 1.13 \end{pmatrix}, \; \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.43 \\ 0.90 \\ 0.66 \end{pmatrix},$$

$$\begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 0.29 \\ 1.16 \\ 3.44 \end{pmatrix}, \; \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 5.52 \\ 11.13 \\ 22.72 \end{pmatrix}, \; \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 0.58 \\ 1.03 \\ 1.53 \end{pmatrix}, \; \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.88 \\ 3.59 \end{pmatrix}, \; \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 2.82 \\ 8.94 \\ 25.90 \end{pmatrix},$$

$$\begin{pmatrix} N_{1e} \\ N_{2e} \\ N_{3e} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.18 \\ 0.52 \end{pmatrix}, \; \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 0.58 \\ 2.48 \\ 7.34 \end{pmatrix}, \; \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 12.89 \\ 27.48 \\ 58.09 \end{pmatrix}, \; \begin{pmatrix} K_{1e} \\ K_{2e} \\ K_{3e} \end{pmatrix} = \begin{pmatrix} 0.76 \\ 2.24 \\ 4.71 \end{pmatrix}, \; \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 6.13 \\ 3.36 \\ 2.15 \end{pmatrix},$$

$$\begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{pmatrix} = \begin{pmatrix} 8.18 \\ 4.58 \\ 3.01 \end{pmatrix}, \; \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1.84 \\ 1.11 \\ 0.76 \end{pmatrix}, \; \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} 3.28 \\ 1.94 \\ 1.33 \end{pmatrix}.$$

It is straightforward to calculate the six eigenvalues at the equilibrium point as follows

$$-0.21, \; -0.19, \; -0.17, \; -0.13, \; -0.11, \; -0.08.$$

The equilibrium point is stable. This conclusion is important as it guarantees that we can effectively carry out comparative dynamic analysis.

4. Comparative Dynamic Analysis

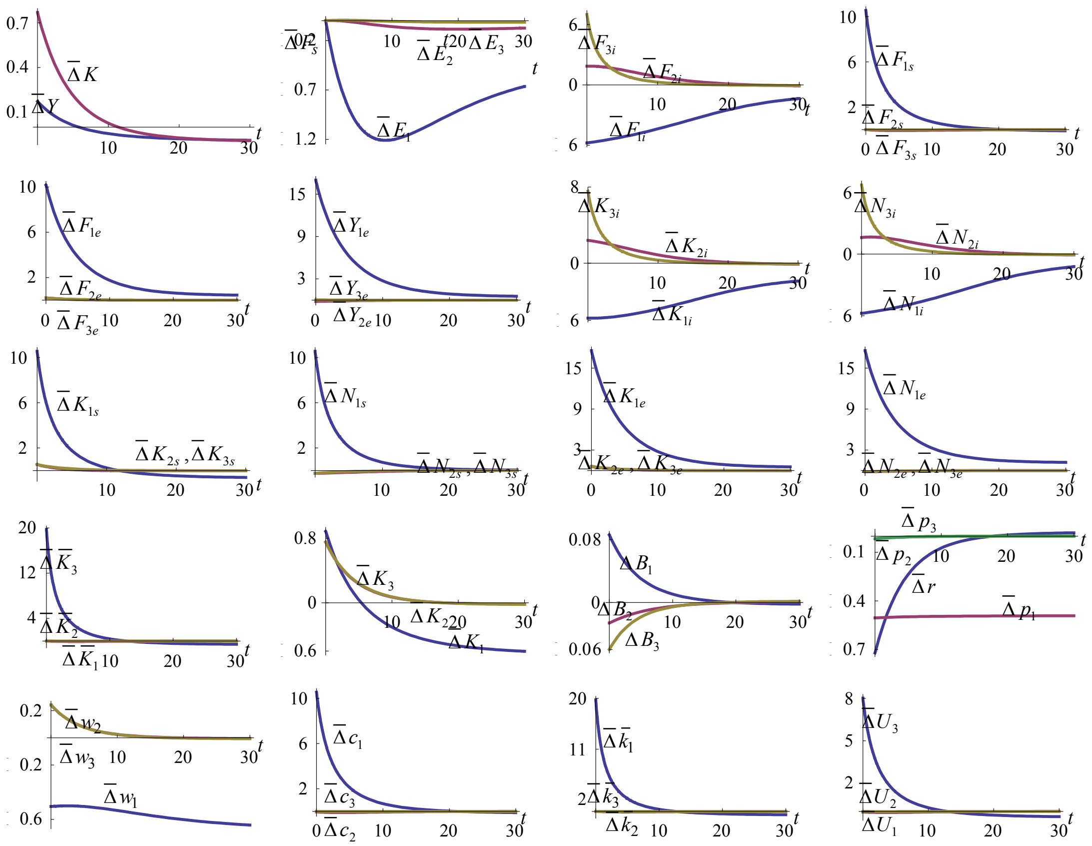
We simulated the motion of the dynamic system. It is important to ask questions such as how changes in one country's environmental conditions will affect the global economy and environment of different countries. First, we introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

4.1 The tax rate on the capital goods sector being increased in the DE

We first study the case when the environmental tax rate on the capital goods sector is increased in the developed economy as follows: $\tau_{1i} : 0.01 \Rightarrow 0.015$.

The simulation result is plotted in Figure 2. The global capital and output levels are increased initially but reduced in the long term. The DE's tax income for improving environment is increased. The rise in the environmental income enlarges the DE's environmental sector. The change in the DE's tax rate changes the economic structure of the global economy. The output level of the DE's capital goods sector is reduced, while the DE's other two sectors are enlarged. The output levels of the other two countries' capital goods sectors are augmented. The consumption level and wealth of the DE are increased initially but are affected slightly in the long term. The DE's environment is improved, while environmental conditions in the other two economies are slightly improved. The rate of interest is reduced in the short term, but is slightly affected in the long term. In the long term, the DE's utility level and wage rate are lowered. The utility levels and wage rates of the two other countries are slightly affected. The effects on the other variables are listed in Figure 1. We see that the change in the tax rate affects all the countries. It should be noted that Our paper deals a closed world economic system with multiple countries. In reality, some open economies are too small to significantly affect other economies. There are a large amount of studies on small open growth literature. As far as a national economy is concerned, our analytical framework is quite similar to the economic growth model in a neoclassical framework proposed by Adu (2013). In Adu's approach pollution is treated as an argument in both the utility function and production function. The utility function is different from our approach, while this study uses the same production function. According to the model by Add, environmental policy imposes a drag on long-run growth. As our framework is more general, our conclusions are more robust than the conclusions from analyzing a single economy.

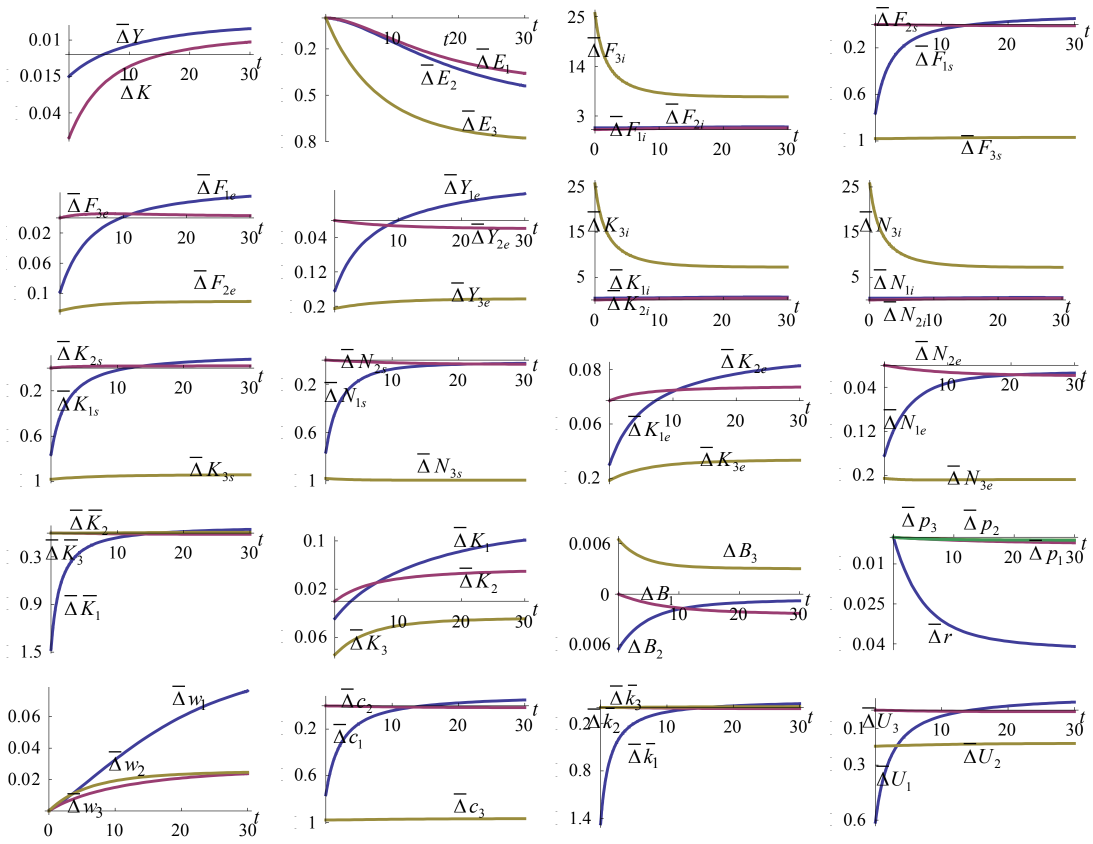
Figure 2. A Rise in the Tax Rate on the Capital Goods Sector in the DE



4.2 The consumption tax rate being increased in the UE

We now study what happens in the global economy if the UE economy raises the consumption tax rate as follows: $\tau_{3c} : 0.01 \Rightarrow 0.02$. The simulation result is plotted in Figure 3. The UD's consumption is reduced, while the ID's consumption is slightly affected. The DE's per capita consumption level is reduced initially, but slightly increased in the long term. The prices of consumer goods and rate of interest are slightly affected in the three economies. The global wealth and output are lowered initially but augmented in the long term. The environment is improved in all the three countries. As the tax rate on consumption is increased, the output level of the UE's consumer goods sector is slightly reduced and the output level of the capital goods sector is increased. The net effect of the tax rate change results in reduction of the tax income in the UE. The output level of the environmental sector is reduced in association with the improved environment and reduced tax income. The prices of consumer goods in the three economies and the rate of interest in the global are slightly affected. The wage rates are enhanced in the three economies. The UE's trade balance is slightly improved, while the other two countries' trade balances are slightly deteriorated. The utility level in the UE is reduced, the utility level in the IE is slightly affected, and the utility level in the DE is reduced initially but increased in the long term. The effects on the other variables are plotted in Figure 3.

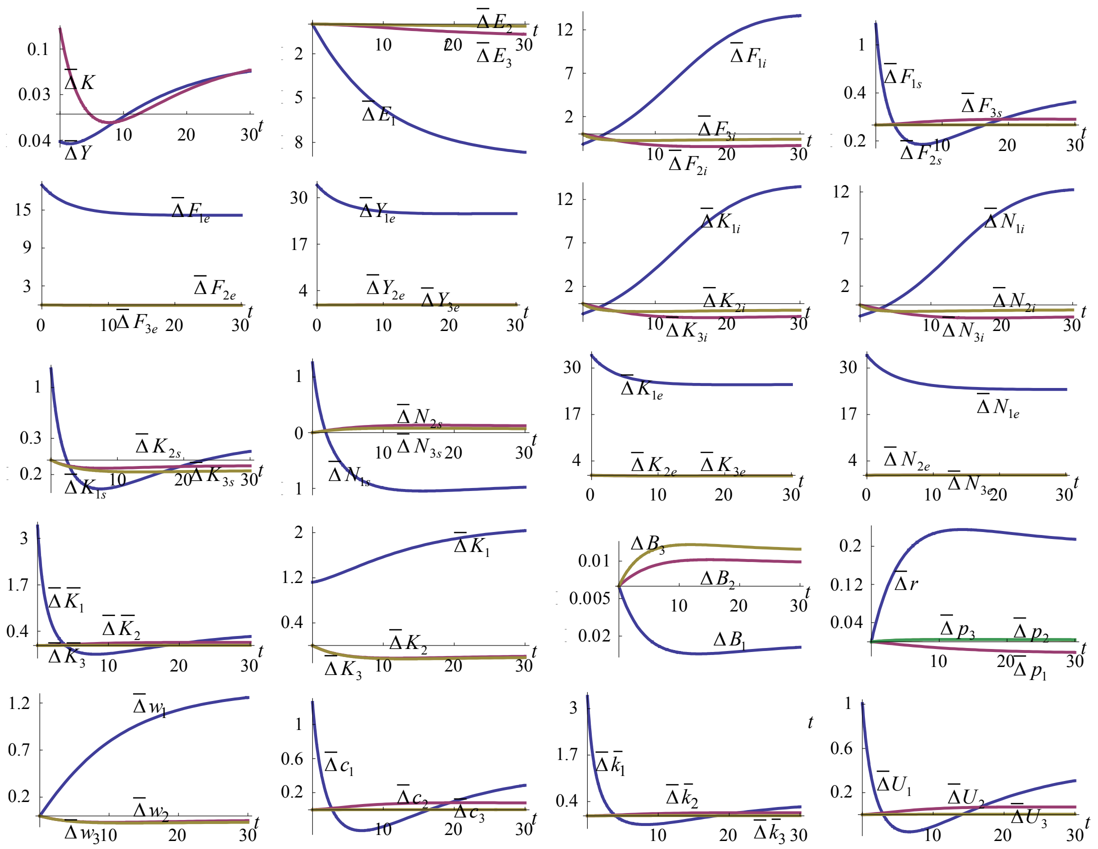
Figure 3. The Consumption Tax Rate Being Increased in the UE



4.3 The tax rate on wage income being increased in the DE

We now study what happens in the global economy if the DE economy raises the tax rate on wage income as follows: $\tau_{1w}: 0.01 \Rightarrow 0.02$. The simulation result is plotted in Figure 4. The DE's wage rate is increased, while the wage rates in the other two countries are lowered. The rate of interest is increased. The change in the global wealth is U-type, while the global income falls initially but rises in the long term. The DE raises the tax income for protecting environment. The output of the environmental sector is increased and environment is improved. Partly because the DE pollutes less the other two countries, environment is improved in the other two countries. The price of consumer goods in the DE falls, while in the other two countries the prices are increased. The change in the DE's utility level is U-type, while the utility levels in the other countries are slightly affected. The DE's trade balance is deteriorated, while the trade balances of the other two countries are improved. The DE uses more the global wealth, while the other two economies less.

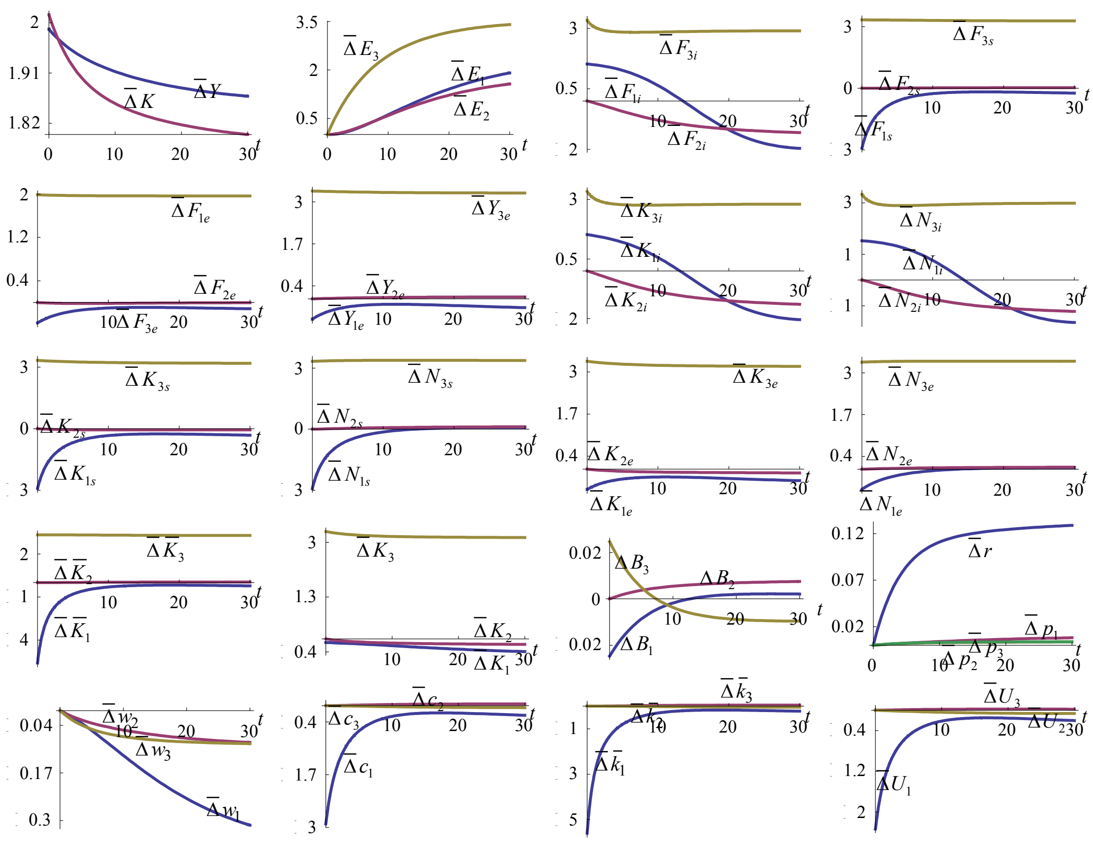
Figure 4. A Rise in the Tax Rate on the DE's Wage Income



4.4 The UE's population being increased

It has been observed that the effect of population growth varies with the level of economic development (e.g., Ehlich and Lui, 1997, Galor and Weil, 1999, Boucekkine, et al., 2002). We now examine effects of changes in the population in a globally connected world with goods and environment. We now consider the case that the DE's population is increased as follows: $N_3 : 30 \Rightarrow 31$. The simulation result is plotted in Figure 5. As the population is increased, environment is deteriorated in all the three economies. We also see that wage rates in the three economies are also reduced. The global wealth and global output are augmented. The IE's trade balance is improved. The DE's trade balance is initially deteriorated but improved in the long term. The UE's trade balance is improved first and then deteriorated. The macroeconomic variables of the UE are all increased because of the population expansion. The utility level in the UE is reduced. The utility levels in the other two economies are slightly affected. The effects on the other variables are plotted in Figure 5.

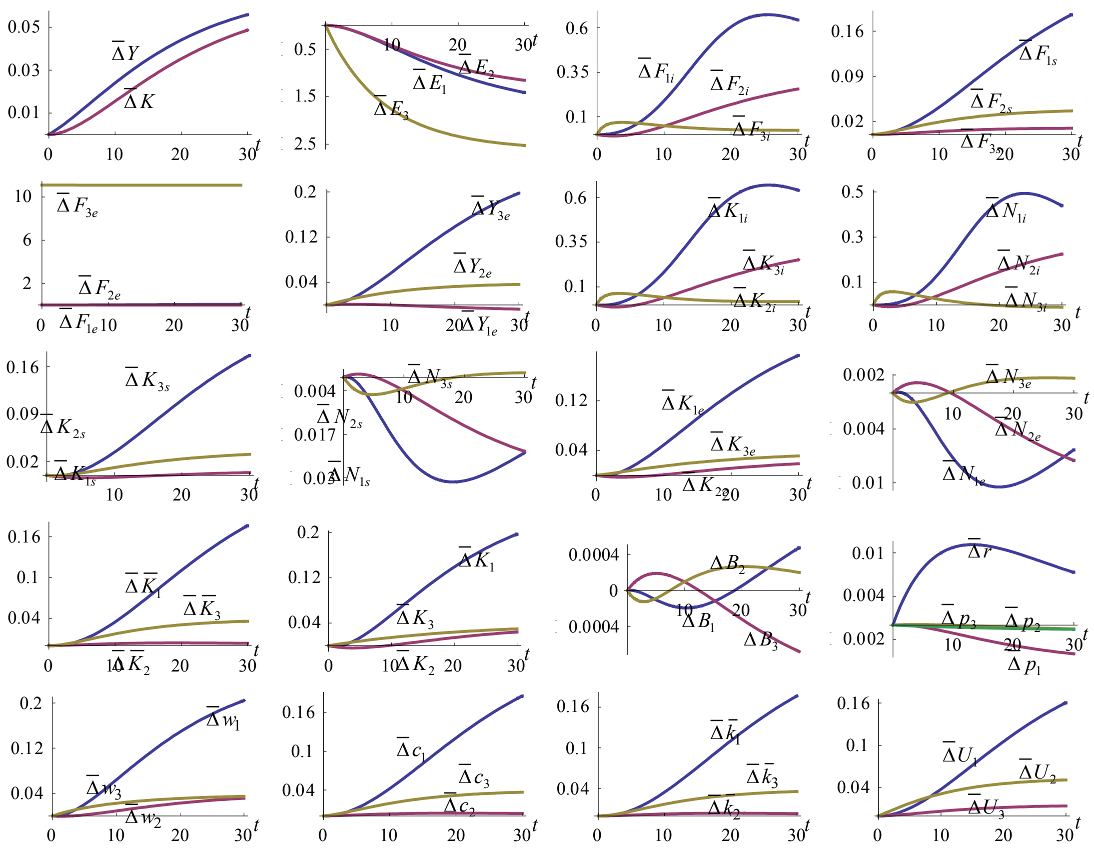
Figure 5. A Rise in the Population of the UE



4.5 The environment sector's productivity in the UE being enhanced

The UE raises the total productivity of the environment sector as follows: $A_{3e} : 0.9 \Rightarrow 1$. The simulation result is plotted in Figure 6. The output of the UE's environment sector is augmented greatly. This results in the improvement in the UE's environment. Partly because the UE pollutes less the other two economies, environment is improved both in the IE and the DE. The wage rates in the three economies and the rate of interest in the global market are slightly increased.

Figure 6. The Productivity of the UE's Environment Sector Being Enhanced



5. Conclusions and Discussion

This paper constructed a multi-country growth model with endogenous physical capital and wealth accumulation and environmental change. The study analyzed not only inequalities in income, wealth and economic structures between countries, but also examined differences in economic growth and environmental changes in each country and international trade and transboundary pollution between countries. This paper makes a unique contribution to the theoretical literature of global growth and environmental change in that it synthesizes the economic mechanisms of growth, international trade and environmental changes on microeconomic foundations within a compact framework. Different from the growth models with the Ramsey approach, we used an alternative utility function to determine saving and consumption. We could comprehensively discuss some important issues related to growth and environmental change in a unique manner because our analytical framework contains not only the economic mechanisms for analyzing these issues, but also because we provided the computational procedure to follow the motion of the nonlinear dynamic system. The comparative analyses provide some insights into the complexity of economic growth with environment. For instance, the study by Grossman and Krueger (1995: 353) identifies no evidence that "environmental quality deteriorates steadily with economic growth." Our simulation indicates that although the conclusion made by Crossman and Krueger holds for some countries, but is invalid for certain countries. We showed that that in association with environmental changes, different countries experience different changes in terms of wage, consumption, and wealth. Our model can be extended and generalized in different directions. It is well known that one-sector neoclassical growth model has been generalized and extended in many directions. There are also different economic models with environmental changes. We may further develop our model on the basis of these lines. It is also important to further analyze behavior of the model with other forms of production or utility functions. There are other possible ways of transboundary pollution (e.g., Copeland and Taylor, 1995, 2003; Ono, 1998; Chao and Yu, 1999; Xing and Kolstad, 2002; Naito, 2003; Javorcik and Wei, 2004; Takarada, 2005; Schweinberger and Woodland, 2008; Suhardiman and Giordano, 2012). Another important extension of the paper is to introduce international negotiations about pollution control.

Appendix: Proving Lemma 1

By (2), (4) and (14), we obtain

$$z_j \equiv \frac{r + \delta_k}{w_j} = \frac{N_{jm}}{\bar{\beta}_{jm} K_{jm}}, \quad j = 1, ..., J, \quad m = i, s, e, \tag{A1}$$

where $\bar{\beta}_{jm} \equiv \beta_{jm} / \alpha_{jm}$. Insert (A1) in (2)

$$r = \alpha_{jr} \Gamma_{ji} z_j^{\beta_{ji}} - \delta_k, \quad w_j = \alpha_j \Gamma_{ji} z_j^{-\alpha_{ji}}, \tag{A2}$$

where

$$\alpha_{jr} = \alpha_{ji} \bar{\tau}_{ji} \bar{\beta}_{ji}^{\beta_{ji}} A_{ji}, \quad \alpha_j = \frac{\beta_{ji} \bar{\tau}_{ji} A_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}}.$$

From (A2) we also have

$$r = \alpha_{jr} \Gamma_{ji} z_j^{\beta_{ji}} - \delta_{jk} = \alpha_{1r} \Gamma_{1i} z_1^{\beta_{1i}} - \delta_{1k}, \quad j = 1, ..., J.$$

From the above equations we solve

$$z_j\left(z_1, \left(E_j\right)\right) = \left(\frac{\alpha_{1r} \Gamma_{1i} z_1^{\beta_{1i}} + \delta_{jk} - \delta_{1k}}{\alpha_{jr} \Gamma_{ji}}\right)^{1/\beta_{ji}}, \quad j = 2, ..., J. \tag{A3}$$

Hence, we determine r, w_j, and z_j, as functions of z_1 and $\left(E_j\right)$. From (3) and (4), we have

$$p_j\left(z_1, \left(E_j\right)\right) = \frac{\bar{\beta}_{js}^{\alpha_{js}} z_j^{\alpha_{js}} w_j}{\beta_{js} \bar{\tau}_{js} A_{js} \Gamma_{js}}. \tag{A4}$$

From (A4) and the definitions of $\hat{y}_j$, we have

$$\hat{y}_j = \left(1 + \bar{\tau}_{jk} r\right)\bar{k}_j + \bar{\tau}_{jw} w_j . \tag{A5}$$

Insert $p_j c_j = \xi_j \hat{y}_j$ in (15)

$$\xi_j N_j \hat{y}_j = p_j F_{js} . \tag{A6}$$

Substituting (A5) in (A6) yields

$$N_{js} = g_j \bar{k}_j + \bar{g}_j , \tag{A7}$$

where we use $w_j N_{js} = \beta_{js} \bar{\tau}_{js} p_j F_{js}$ and

$$g_j\left(z_j , \left(E_j\right)\right) \equiv \left(\frac{1 + \bar{\tau}_{jk} r}{w_j}\right)\xi_j \beta_{js} \bar{\tau}_{js} N_j , \quad \bar{g}_j \equiv \bar{\tau}_{jw} \xi_j \beta_{js} \bar{\tau}_{js} N_j .$$

From (A1) and (16), we get

$$\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{js}}{\bar{\beta}_{js}} + \frac{N_{je}}{\bar{\beta}_{je}} = z_j K_j . \tag{A8}$$

Insert (A7) in (A8)

$$\frac{N_{ji}}{\bar{\beta}_{ji}} + \frac{N_{je}}{\bar{\beta}_{je}} = z_j K_j - \frac{g_j \bar{k}_j}{\bar{\beta}_{js}} - \frac{\bar{g}_j}{\bar{\beta}_{js}}. \tag{A9}$$

Insert (A7) in $N_{ji} + N_{js} + N_{je} = N_j$

$$N_{ji} + N_{je} = N_j - g_j \bar{k}_j - \bar{g}_j . \tag{A10}$$

Solve (A9) and (A10) with N_{ji} and N_{js} as the variables

$$\begin{aligned} N_{ji} &= a_{ji} + \tilde{b}_{ji} \bar{k}_j - \bar{\beta}_j z_j K_j , \\ N_{je} &= a_{je} + \tilde{b}_{je} \bar{k}_j + \bar{\beta}_j z_j K_j , \end{aligned} \tag{A11}$$

where

$$a_{ji} \equiv \left(\frac{N_j - \bar{g}_j}{\bar{\beta}_{je}} + \frac{\bar{g}_j}{\bar{\beta}_{js}}\right)\bar{\beta}_j , \quad \tilde{b}_{ji}\left(z , \left(E_j\right)\right) \equiv \left(\frac{1}{\bar{\beta}_{js}} - \frac{1}{\bar{\beta}_{je}}\right)\bar{\beta}_j g_j ,$$

$$a_{je} \equiv -\left(\frac{\bar{g}_j}{\bar{\beta}_{js}} + \frac{N_j - \bar{g}_j}{\bar{\beta}_{ji}}\right)\bar{\beta}_j, \quad \tilde{b}_{je}\left(z, \left(E_j\right)\right) \equiv \left(\frac{1}{\bar{\beta}_{ji}} - \frac{1}{\bar{\beta}_{js}}\right)\bar{\beta}_j\, g_j, \quad \bar{\beta}_j \equiv \left(\frac{1}{\bar{\beta}_{je}} - \frac{1}{\bar{\beta}_{ji}}\right)^{-1}.$$

Substituting (A1) into (2) and (5) yields

$$F_{ji} = \frac{A_{ji}\,\Gamma_{ji}\,N_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}\, z_j^{\alpha_{ji}}}, \quad F_{js} = \frac{A_{js}\,\Gamma_{js}\,N_{js}}{\bar{\beta}_{s}^{\alpha_{s}}\, z_j^{\alpha_{js}}}. \tag{A12}$$

Insert (A12) in (12)

$$Y_{je} = \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \tau_{jc}\,c_j\,N_j + \tau_{jw}\,w_j\,N_j + \tau_{jk}\,r\,\bar{k}_j\,N_j. \tag{A13}$$

where

$$\Lambda_{ji}\left(z_1, \left(E_j\right)\right) \equiv \frac{\tau_{ji}\,A_{ji}\,\Gamma_{ji}}{\bar{\beta}_{ji}^{\alpha_{ji}}\, z_j^{\alpha_{ji}}}, \quad \Lambda_{js}\left(z_1, \left(E_j\right)\right) \equiv \frac{\tau_{js}\,A_{js}\,\Gamma_{js}}{\bar{\beta}_{js}^{\alpha_{js}}\, z_j^{\alpha_{js}}}.$$

From $p_j\,c_j = \xi_j\,\hat{y}_j$ and (A5), we have

$$c_j = \left(\frac{1 + \bar{\tau}_{jk}\,r}{p_j}\right)\xi_j\,\bar{k}_j + \frac{\bar{\tau}_{jw}\,\xi_j\,w_j}{p_j}. \tag{A14}$$

Substituting (A14) into (A13) yields

$$Y_{je} = \bar{\Lambda}_j + \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \Lambda_j\,\bar{k}_j, \tag{A15}$$

where

$$\Lambda_j\left(z_1, \left(E_j\right)\right) \equiv \left(\frac{1 + \bar{\tau}_{jk}\,r}{p_j}\right)\xi_j\,\tau_{jc}\,N_j + \tau_{jk}\,r\,N_j, \quad \bar{\Lambda}_j\left(z_1, \left(E_j\right)\right) \equiv \left(\frac{\bar{\tau}_{jw}\,\xi_j\,\tau_{jc}}{p_j} + \tau_{jw}\right)w_j\,N_j.$$

Insert (A15) in $w_j\,N_{je} = \beta_{je}\,Y_{je}$

$$\frac{w_j\,N_{je}}{\beta_{je}} = \bar{\Lambda}_j + \Lambda_{ji}\,N_{ji} + \Lambda_{js}\,N_{js} + \Lambda_j\,\bar{k}_j. \tag{A16}$$

Substituting (A7) and (A11) into (A16) yields

$$K_j = \bar{\Delta}_j + \Delta_j\,\bar{k}_j, \tag{A17}$$

where

$$\bar{\Delta}_j\left(z_1, \left(E_j\right)\right) \equiv \left(\bar{\Lambda}_j + \Lambda_{js}\,\bar{g}_j - \frac{w_j\, a_{je}}{\beta_{je}} + a_{ji}\,\Lambda_{ji}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1} \frac{1}{\bar{\beta}_j\, z_j},$$

$$\Delta_j\left(z_1, \left(E_j\right)\right) \equiv \left(\tilde{b}_{ji}\,\Lambda_{ji} + g_j\,\Lambda_{js} + \Lambda_j - \frac{w_j\,\tilde{b}_{je}}{\beta_{je}}\right)\left(\frac{w_j}{\beta_{je}} + \Lambda_{ji}\right)^{-1} \frac{1}{\bar{\beta}_j\, z_j}.$$

Insert (A17) in (17)

$$\sum_{j=1}^{J} \bar{\Delta}_j + \sum_{j=1}^{J} \Delta_j\, \bar{k}_j = \sum_{j=1}^{J} \bar{k}_j\, N_j. \tag{A18}$$

Solve (A18) with $\bar{k}_1$ as the variable

$$\bar{k}_1 = \varphi\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \left(\sum_{j=1}^{J} \bar{\Delta}_j + \sum_{j=2}^{J}\left(\Delta_j - N_j\right)\bar{k}_j\right)\frac{1}{N_1 - \Delta_1}. \tag{A19}$$

where $\left\{\bar{k}_j\right\} \equiv \left(\bar{k}_2, \ldots, \bar{k}_J\right)$.

It is straightforward to confirm that all the variables can be expressed as functions of z_1, $\left(E_j\right)$, and $\left\{\bar{k}_j\right\}$ by the following procedure: r and w_j by (A2) $\rightarrow$ p_j by (A4) $\rightarrow$ $\bar{k}_1$ by (A19) $\rightarrow$ K_j by (A17) $\rightarrow$ N_{ji} and N_{je} by (A11) $\rightarrow$ N_{js} by (A7) $\rightarrow$ K_{je}, K_{js}, and K_{ji} by (A1) $\rightarrow$ $\hat{y}_j$ by (A5) $\rightarrow F_{ji}$, F_{js} and F_{je} by the definitions $\rightarrow$ c_j and s_j by (10) $\rightarrow$ $Y_{je} = w_j\, N_{je} / \beta_{je} \rightarrow K = \sum_j \bar{k}_j\, N_j \rightarrow \bar{K}_j = \bar{k}_j\, N_j \rightarrow B_j = \left(\bar{K}_j - K_j\right) r$ $\rightarrow U_j$ by the definitions. From this procedure, (A19), (5) and (11), we have

$$\dot{\bar{k}}_1 = \bar{\Phi}_1\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \lambda_1\, \hat{y}_1 - \varphi, \tag{A20}$$

$$\dot{\bar{k}}_j = \Phi_j\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \lambda_j\, \hat{y}_j - \bar{k}_j, \quad j = 2, \ldots, J.$$

$$\dot{E}_j = \Omega_j\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \theta_{ji}\, F_{ji} + \theta_{js}\, F_{js} + \theta_j\, C_j - F_{je} - \bar{\theta}_j\, E_j + \Omega_j\left(\left(E_q\right)\right). \tag{A21}$$

Taking derivatives of equation (A19) with respect to t and combining with (A21) implies

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z_1}\dot{z}_1 + \sum_{j=1}^{J} \Omega_j \frac{\partial \varphi}{\partial E_j} + \sum_{j=2}^{J} \Phi_j \frac{\partial \varphi}{\partial \bar{k}_j}. \tag{A22}$$

Equaling the right-hand sizes of equations (A20) and (A22), we get

$$\dot{z}_1 = \Phi_1\left(z_1, \left(E_j\right), \left\{\bar{k}_j\right\}\right) \equiv \left[\bar{\Phi}_1 - \sum_{j=1}^{J} \Omega_j \frac{\partial \varphi}{\partial E_j} - \sum_{j=2}^{J} \Phi_j \frac{\partial \varphi}{\partial \bar{k}_j}\right]\left(\frac{\partial \varphi}{\partial z}\right)^{-1}. \tag{A23}$$

In summary, we proved the lemma.

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