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Formulating a Stochastic Discounting Model with Actuarial and Risk Management Applications

By

Constantinos T. Artikis
National and Kapodistrian University of Athens, Faculty of Primary Education,
Department of Mathematics and Informatics,
20, Ippokratus str. Athens, Greece, Email: ctartikis@gmail.com

Abstract

Stochastic discounting models are generally recognized as extremely strong analytical tools for a very wide variety of fundamental areas in the actuarial discipline. The paper is mainly devoted to the formulation, investigation and application in the actuarial discipline of a stochastic discounting model. It is shown that the formulated stochastic discounting model can substantially support the role of proactivity in making insurance decisions.

JEL Classification: C51.

Keywords: Stochastic Discounting, Risk Management, Model.

1. Introduction

Fundamental decision making processes identify stochastic discounting models as extremely strong tools for describing and solving problems of particular importance. More precisely, such models are very useful for a wide variety of practical disciplines. Economics, management, operational research and insurance are generally recognized as significant practical disciplines making extensive use of stochastic discounting models. These models make use of the results of probability theory and provide actuaries, risk analysts, risk managers and other risk experts with valuable information for developing criteria suitable in making realistic decisions. Principal among the methods for investigating properties of a stochastic discounting model is the derivation of the corresponding distribution function. It is generally recognized that the derivation of such a distribution function is a very difficult problem (Dall' Aglio, 1964, De Schepper *et al.*, 1982, Bühlmann, 1992, Aebi *et al.*, 1994, Parker, 1994, Jerwood and Moshakis, 1997, Parker, 1998, Casey, 2001). During the last decade, many research contributions have been concentrated on the formulation, investigation, and applications of stochastic discounting models with particular practical importance (Booth, 2003, Hoedemakers *et al.*, 2005, Ahcan *et al.*, 2006, Ermoliev *et al.*, 2008, Wu-Li, 2009, Asimit and Batescu, 2010, Rasmussen, 2010,Wang *et al.*, 2010, Plunovskly and Zhang, 2011, Righter, 2011, Leveille and Adekambi, 2012, Chretien, 2012). Recently, fundamental concepts of extreme value theory have been used in stochastic discounting modelling activities (Artikis, P. and Artikis, C., 2005, 2007, Artikis *et al.*, 2007, Artikis *et al.*, 2008, Artikis, P. and Artikis, C., 2009).

The present paper concentrates on the implementation of three purposes. The first purpose is the formulation of a stochastic discounting model by making use of a sequence of nonnegative random variables, a discrete random variable, two nonnegative random variables, and a Bernoulli random variable. The second purpose is the establishment of sufficient conditions for embedding the characteristic function of the formulated stochastic discounting model into the class of characteristic functions corresponding to stochastic multiplicative models, based on two nonnegative and independent random variables. The third purpose is the interpretation of the formulated stochastic discounting model as a very useful actuarial tool for making insurance decisions. It is shown that the formulated stochastic discounting model can provide the discipline of insurance with the fundamental elements of proactive risk treatment operations.

2. Formulation of a Stochastic Discounting Model

The present section of the paper concentrates on the formulation of a stochastic discounting model by making use of a discrete random variable, a sequence of nonnegative, and identically distributed random variables, two nonnegative random variables and a Bernoulli random variable. More precisely, the paper provides an extension of a known stochastic discounting model (Jerwood and Moshakis, 1997).

We suppose that N is a discrete random variable taking values in the set

$$N_0 = \{0,1,2,\ldots\} \tag{1}$$

and probability generating function

$$P_N(z) \tag{2}$$

We also suppose that

$$\{X_n, n = 1, 2, ...\} \tag{3}$$

is a sequence of nonnegative and independent random variables distributed as the random variable *X* with characteristic function:

$$\varphi_X(u) \tag{4}$$

We consider the random sum

$$S = X_1 + X_2 + ... + X_N \tag{5}$$

Moreover, we consider the nonnegative random variable *Y* with characteristic function:

$$\varphi_Y(u) \tag{6}$$

the Bernoulli random variable *C* with probability generating function

$$P_C(z) = pz + q\ ,\ 0 < p < 1\ ,\ q = 1 - p\ , \tag{7}$$

the nonnegative random variable *T* with distribution function

$$F_T(t) \tag{8}$$

and the nonnegative real number *r*. We consider the random variable

$$\mathrm{L} = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases} \tag{9}$$

and the stochastic model

$$V = Le^{-rT} \tag{10}$$

The above stochastic model can be interpreted as an analytical tool of continuous discounting in the following way.

We suppose that the random variable N denotes the number of cash flows arising at the random time point T and the random variable

$$X_n, n = 1, 2, \ldots \tag{11}$$

denotes the size of the n th cash flow. Hence the random sum

$$S = X_1 + X_2 + \ldots + X_N \tag{12}$$

denotes the total size of the N cash flows arising at the random time point T with probability p. We also suppose that the random variable Y denotes the size of a cash flow arising at the random time point T with probability q.

It is easily seen that the random variable

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases} \tag{13}$$

denotes the size of the cash flow arising at the random time point T.

If the nonnegative real number r denotes force of interest then the stochastic model

$$V = Le^{-rT} \tag{14}$$

denotes the present value, as viewed from the time point 0, of the cash flow

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases} \tag{15}$$

arising at the random time point T.

The presence of the Bernoulli random variable C and the presence of the random sum

$$S = X_1 + X_2 + \ldots + X_N \tag{16}$$

in the mathematical structure of the stochastic discounting model

$$V = Le^{-rT} \tag{17}$$

constitute very good reasons for investigating the role of the corresponding characteristic function

$$\varphi_V(u) \tag{18}$$

in considering applications in different practical disciplines of this stochastic discounting model.

3. Characteristic Function of a Stochastic Discounting Model

The present section of the paper concentrates on the evaluation of the characteristic function of the formulated stochastic discounting model since an evaluation of the corresponding distribution function is not possible. More precisely, the section establishes conditions for embedding such a characteristic function into an important class of characteristic functions.

If N, $\{X_n, n = 1, 2, \ldots\}$, Y, C, T are independent then it follows that the random variables Y, C, T, are independent, N, $\{X_n, n = 1, 2, \ldots\}$ are independent, the random variables $S = X_1 + X_2 + \ldots + X_N$, T, C are independent and that the random variables

$$L = \begin{cases} Y & , \quad C = 0 \\ S & , \quad C = 1 \end{cases}$$

, T are also independent. Hence the random variables L, $W = e^{-rT}$ are independent and it can be easily demonstrated that

$$\varphi_V(u) = \int_0^1 \left[q\varphi_Y(uw) + pP_N(\varphi_X(uw))\right] d\left[1 - F_T\left(-\frac{1}{r}\log w\right)\right] \tag{19}$$

is the characteristic function of the stochastic discounting model

$$V = Le^{-rT}. \tag{20}$$

It is quite obvious that the theoretical contribution of the present section of the paper consists of establishing sufficient conditions for embedding the formulated stochastic discounting model into the class of stochastic multiplicative models, based on two nonnegative and independent random variables (Keilson and Steutel, 1974). Such a contribution substantially facilitates the applicability of the formulated stochastic discounting model in a very wide variety of practical disciplines.

4. Actuarial and Risk Management Applications of a Stochastic Discounting Model

The present section of the paper concentrates on the interpretation of the formulated stochastic discounting model as a strong actuarial tool for making risk financing decisions.

We suppose that the discrete random variable N denotes a number of risks at the random time point T and the nonnegative random variable

$$X_n, \; n = 1,2,... \tag{21}$$

denotes the severity of the n th risk. Hence the random sum

$$S = X_1 + X_2 + ... + X_N \tag{22}$$

denotes the severity of the N risks at the random time point T.

We also suppose that the nonnegative random variable Y denotes the severity of another risk at the random time point T.

Moreover, we suppose that the N risks with severity

$$S = X_1 + X_2 + ... + X_N \tag{23}$$

can be undertaken with probability p by an insurance company at the random time point T or the risk with severity Y can be undertaken with probability q by the same insurance company at the random time point T.

Hence the random variable

$$L = \begin{cases} Y & , \quad C=0 \\ S & , \quad C=1 \end{cases} \tag{24}$$

denotes the obligation undertaken by the insurance company at the random time point T and the stochastic discounting model

$$V = Le^{-rT} \tag{25}$$

the present value of this obligation, as viewed from time point 0. It is easily seen that the actuarial interpretation of the formulated stochastic discounting model, established by the present section of the paper, provides risk analysts, risk managers and other risk experts with valuable probabilistic information for insurance decision making.

5. Conclusions

The formulation and investigation of a stochastic discounting model constitute the theoretical contribution of the paper. Moreover, the practical contribution of the paper consists of the interpretation of the formulated stochastic discounting model as a strong actuarial tool for making and implementing insurance decisions in a proactive manner. It is shown that the formulated stochastic discounting model can support the presence of proactivity in insurance activities.

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