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Parliamentary Coalitions, An n-person Game Approach to Politics

By

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Abstract

In this paper, we present the general contribution of n-person game in turbulent environment of parliamentary coalitions. Same basic data about the coalition form and the characteristic function is necessary in order to connect n-person game theory and behavioral game theory. Taking the Norway elections as an example we study the possibility of a required long term coalition in Greece. We potentially suggest which parties could form a coalition by using game theory for those cases, where the choice of one party government is not possible.

JEL Classifications: C, C7, C71.

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1. Introduction

Game theory could be described as the decision theory of n-players, where each player's choices affect the performance of others, something that players take into account before any decision (Camerer, 2003). In this paper we consider the contribution of game theory to design scenarios in order to analyze and understand the behavior of people engaged in strategic interaction.

Analyzing some economic systems in our daily life such as the farmers' market or daily transportation, we understand that all of us take decisions under complete or incomplete information. For example at the farmers' market, a farmer on the bench thinking about the price of his tomatoes in view of customer traffic and competition. Customer is trying to decide; to buy from the first bench, to search more or to wait.

All these decisions interact to determine the effect of commodity prices and ultimately to buy the products we want. Another economic system it the transportation, where we are asked to make decisions about the path or the type of transport we choose to move. The decision we take will affect the decision of others and eventually through this interaction leads to equilibrium. The same is happened in parliamentary democracies with more than two major parties which should explore the possibility of forming a coalition government.

2. N-person game

Firstly we will give some information on n-people games. A zero-person game is a mechanical model or a behaviorist model if involves human factor. One person game is a standard decision problem with perhaps nature, as a non-player, personifying the element of uncertainty faced by the decision maker. A game with two or more players is a quite different form of uncertainty, which seems to be due to the exercise of free choice by independent agents. With three or more players, the coalition formed becomes an important and sometimes decisive chance, and here is the typical field of game theory n-persons including multilateral decisions models (international trade, elections, markets) (Shapley, 1968). The various interests in an n-person game are at cross-purposes. The parallel interests (as in theory of teams) or direct opposed interests (as in 2-persons game, zero-sum) tend to wipe out the coalition question and hence to permit the more explicit methods of direct optimization and minimax. N-persons game theory finds wide application in areas such as cooperation, coalition, organizational structure, commitment, trust, compromise, threat and enforceability (Shapley, 1968).

The frame of the characteristic function of a game is a fundamental idea of von Neumann (von Neumann and Morgenstern, 1953). The characteristic function points out a numerical value on each potential coalition of players and virtually removes details such as information, timing, payoffs and moves, while the threat of the n-person problem is a standalone system within the tank of all strategic distractions (Shapley, 1968). As Shapley mentions the characteristic function is not always enough for the different types of games and there are at least two important conditions for its use. Firstly, the payoff product must be clear and explicit otherwise the potential of a coalition could be possibly lost its tonnage in order to represent a freely sharable utility. The second condition has to do with the threats. Threats, including the cost that should be carried out, should not be a determining factor in the coalition reflection. At this point the characteristic function pessimistically assumes that its coalition will experience the most damaging countermoves by the rest of the players; yet costly threats are always negotiable. Taking the characteristic function into consideration you are just closer than you was in the begging. You can use it as a descriptive or classifying tool in order to approach a pre-solution. Till today many approaches of solutions of a game have been devised and a pluralistic theory has arisen; each solution concept, in its own way, addresses same aspect of the n-person problem (Shapley, 1968).

Nash mentions, in order to define the n-person game that each player has a finite set of pure strategies and in which a definite set of payments to the n-players. That is of course corresponds to each n-tuple of pure strategies, one for each player. Also for mixed strategies, the payoff functions are the expectations of the players, thus becoming polylinear forms in the probabilities with which the players select their pure strategies. Following the above one strategy for each player, selected from the n-tuple tank of strategies may be regarded as a point in the product tank obtained by multiplying the n strategies tank of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the n-1 strategies of the other players in the countering n-tuple. A self-countering n-tuple is called an equilibrium point (Nash, 1950).

2.1 Coalition Form, Characteristic Function

As Ferguson describes (Ferguson, 2005); let $k \geq 2$ denote the number of players in the game, numbered from 1 to k, and let K denote the set of players, $K=\{1,2,....,k\}$. A coalition S, is defined to be a subject of K,S $\subset$ K , and the set of all coalitions is denoted by 2^K. By convention, we also speak of the empty set, $\varnothing$, as a coalition, the empty coalition. The set K is also a coalition, called *grand coalition*. If there are just two players, $k=2$, then there are four coalitions, $\{\{\varnothing\},\{1\},\{2\},K\}$. If there are three players, there are eight coalitions, $\{\{\varnothing\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},K\}$. For n players the set of coalitions 2^K, has 2^k elements.

Definition. The coalition form of an n -person game is given by the pair (N,u), where $N=\{1,2, ..., n\}$ is the set of players and u is the real-value function (subsets of N), and satisfying:

i. $u(\varnothing)=0$, and

ii. If S and T are disjoint coalitions $S \cap T=0$, then $u(S)+u(T) \leq u(S \cup T)$.

The quantity $u(S)$ is a real number for each coalition $S \subset N$, which may be considered as the value of coalition S when its members act together as a unit. Recall that the strategic form of an n-person game is given by the 2n-tuple, $(X_1, X_2, ..., X_n, u_1, u_2, ..., u_n)$, where:

(1). for $i=1, ..., n$, X_i is the set of pure strategies of player i

(2). for $i=1, ..., n$, $u_i(x_1, ... ,x_n)$ is the payoff function to player i, if player 1 uses

$x_1 \in X_1$, player uses $x_2 \in X_2$, and player n uses $x_n \in X_n$.

Transforming a game form strategic form to coalition form entails specifying the value $u(S)$, for each coalition $S \in 2^K$ as the value of the 2-person zero-sum game obtained when the coalition S acts as one player and the complementary coalition, $\overline{S} = K - S$, acts as the other player, and where the payoff to S is the sum of the payoffs to the players in $S: \sum_{i \in S} u_i(x_1,...,x_n)$. Thus:

$$u(S) = Val \left(\sum_{i \in S} u_i\,(x_1,...,x_n) \right) \tag{1}$$

Where the players in S jointly choose x_i for $i \in S$, and the players in $\overline{S}$ chose the x_i for $i \notin S$. The value, $u(S)$, is the analogue of the safety level for coalition S. It describes the total amount that coalition S can have for itself, even if the members of $\overline{S}$ gang up against it, and have as their only object to keep the sum of the payoffs to members of S as small as possible. This is a lower bound to the payoff S should receive because it assumes that the members of $\overline{S}$ ignore what possible payoffs they might receive as a result of their actions.

Example. As Scarf describes (Scarf, 1973), we have a situation of exchange involving three consumers with utility functions $u_1(x)$, $u_2(x)$, $u_3(x)$ and with vectors of initial holdings φ_1, φ_2, φ_3. In order to describe the game in characteristic form, the set of achievable utility vectors must be given for each of the seven possible coalitions – the set of all players, the three two-player coalition, and three coalitions each consisting of a single player. Let's consider the coalition of all three players whose total assets $\varphi^1+\varphi^2+\varphi^3$ can be allocated in an arbitrary fashion x^1, x^2, x^3 among the three members subject only to the constrain $x^1+x^2+x^3 \leq \varphi^1+\varphi^2+\varphi^3$. For any such allocation, the utility triple u_1, u_2, u_3, with $u_i=u_i(x^i)$ for $i=1,2,3$, is obtained: the set of all achievable utility vectors, which we denote by $V_{(123)}$, is then generated by letting the allocations range over all of those consistent with the initial endowment of the coalition. For the coalition (1,2) of the first two players, the set $V_{(12)}$ of achievable utility vectors consists of those utility pairs (u_1, u_2), with $u_1 \leq u_1(x^1)$, and $u_2 \leq u_2(x^2)$ for some x^1, x^2 (with $x^1+x^2 \leq \varphi^1+\varphi^2$), and similarly for each of the remaining two-player coalitions. Each of these sets is contained in the hyperplane whose coordinates corresponding to players not in the coalition are zero. Finally, the single-player coalition have no strategic possibilities available to them, and set $V_{(i)}$ for $i=1,2,3$, may be defined as the set of all points on i-axis not larger than the utility of the i-th player's initial holdings. This example of a three-person exchange economy illustrates the description of a game characteristic form in terms of the set of achievable utility vectors Vs for each coalition of players.

3. Parliamentary coalitions

In a parliamentary democracy with more than two major parties, it is common that no single party will gain the majority of seats in the parliament. Hence a majority government must be formed by a coalition of parties. In this paragraph we will consider several game theoretic approaches to the study of parliamentary coalitions in two courtiers within Europe. Straffin worked on this area and presented the results of 1965 parliamentary elections in Norway (Straffin, 2006). We are going to present what happened in Greek parliamentary elections in Nov. '89, and after that, working on the basis of voting intension surveys that took place this month (Jan. '13), we will place the potentially results and the choices.

3.1 Norway 1965

Parliamentary elections in Norway gave the following results about the number of seats for each party: {A}:68, {B}:13, {C}:18, {D}:18, {E}:31, total 148 seats. It takes 75 members to form a coalition government. Straffin asked if we can predict which parties formed the government. There are five potentially coalitions which are typically winners after they have managed to gain a significant number of seats which gives them simultaneously parliament entering. Since it would not be desirable to have a large number of parties in a government of cooperation, we attempt to predict the possible coalitions that can arise from potential collaborations, resulting in the following: {AB}, {AC}, {AD}, {AE}, {BCDE}. This provision is well known in political science as "Riker size principle" because of William Riker. While there are certainly exceptions to the principle, Riker laying the work to support the theory of political coalitions (Straffin, 2006).

In this case, there are a large number of coalitions which can lead to government. If we want to make a more specific prediction about who they are, there are at least two ways in which the idea of Riker analyzing. Firstly, we can assume that the government is in proportion to the number of votes that the parties of the ruling coalition have taken. For example if we assume that the coalition {AB} is the one who forms the government then {A} takes percentage 68/81, while {B} 13/81. Alternatively, if the coalition government is the {AC} then {A} receives 68/86 and {C} 18/86. We observe that {A} coalition would prefer the percentage 68/81 instead of 68/86, therefore would prefer a smaller combination as {B}, instead of {C}. The parties wish to cooperate with parties that have received as less as possible votes, since this maximizes their share of the coalition. In this example, we might predict that the combination {BCDE} is the most likely form coalition with {AB} as the second most likely. So we have {AB}:81, {AC}:86, {AD}:86, {AE}:99, {BCDE}:80. Secondly, assuming that a government makes equivalent the two members together, it can be argued that all members are equally important. In this case the parties maximize their share of creating a coalition with as few members. We could predict that combinations {AB}, {AC}, {AD}, {AE}, are more likely than the {BCDE}. The above two cases of the principle (Riker) contradict each other in this example, and in fact none of them finds full implementation in practice. This is because there are different parties that ideology would be too difficult to work together and create a coalition government. In this example we can place the five parties on a similar graph as created by Converse and Valen, in relation to the policy approach to economic issues.

TABLE 1

Liberal vs Conservative

	A		B	C	D	E	
Liberal	-5	0	4	6	7	11	Conservative

Axelrod (1970), concerning the above, suggested that governing coalitions which form should be connected, in the sense that it should include all parties in a time line. In this example, the {AB} and {BCDE} are the minimum possible coalition's victory.

The combination of {AC} is not connected, since it does not include {B}, which is contained in any interval containing {A} and {C}. The provision of a minimum connected winning coalition has a reasonable degree of empirical support. Working on this area, de Swaan found that more than half of the 108 coalition governments were minimal winning and connected (de Swaan, 1973). Sometimes the political strategies of parties not identified totally with the political ideologies. For example, a party which is liberal on social issues may follow conservative policies in economics. For this reason, Converse and Valen had a corresponding graph having as basis the political parties in Norway for cultural issues of the country (Converse and Valen, 1971).

TABLE 2

Liberal vs Conservative

	C	E	A	D	B	
Liberal	1	2	3	6	10	Conservative

In this case {AE} and {AD} are the minimum possible coalition's victory. Using a two-dimensional Cartesian system (x: economic issues, y: cultural issues), with the help of Euclidean distance we will try to calculate the relative ideological closeness of the parties.

$$d(A,B)=\sqrt{(-5-4)^2+(3-10)^2}=\sqrt{130}\approx 11.4 \tag{2}$$

$$d(A,C)=\sqrt{(-5-6)^2+(3-1)^2}=\sqrt{125}\approx 11.2 \tag{3}$$

So we can see that party C is slightly closer to A than to party B. Each party wants to be part of a coalition that adopts a platform close to the point of its ideological frame. When we have parties represented as points in an n-dimensional ideological space, we can try to use game-theoretic reasoning to predict which parties will form a governing coalition.

Since Aumann - Maschler bargain sets are most directly tied to coalition structure, it would seem most natural to use that approach. However in a spatial game with ideologically concerned parties, the most natural object of bargaining is not division of spoils. Instead the parties bargain about what kind of policies the government the form will pursue. These platforms can also be represented as points in ideological space. Each party would like to be part of a coalition which adopts a platform close to that party's ideological point. It is thus points in ideological frame, rather than n-tuples of payoffs, which are the basis of offers, objections and counter-objections. Straffin (Straffin, 2006) gave an example in order to explain how the bargaining set idea works in this context.

3.2 Greece

In the history of modern Greece the coalition governments are not many. In order to present the Greek electoral system we will use the information of the general elections of November '89 and after that we will present, according to the last voting intension survey (Jan. '13), the status of voting results from this survey and the frame of the choices.

3.2.1 November 1989

The coalition between New Democracy (ND) and Synaspismos in June 1989 ends on October 1989. The announced elections for November 1989 gives to ND the first place again (46.19%), but still without, for second time, an absolute majority in parliament. The three political leaders, K. Mitsotakis, A. Papandreou and Ch. Florakis agreed to set up a government coalition under the academic, Prof. X. Zolotas. Despite the overwhelming majority of 297 seats available in the parliament, the period of this government characterized by anarchy, and as a result the country was threatened by a deep economic crisis. This was the last coalition government, which became known as Zolotas' party government, which was formed in November 1989, with a lifetime until February 1990. In order to present the possible combinations of political coalitions we place the following tables. The table below shows the number of seats each political party based on official data.

TABLE 3

Party and number of seats

Party	Number of seats
A. ND	148
B. PASOK	128
C. SYNASPISMOS	22
D. OIKOLOGOI	1
E. EMPISTOSYNI	1
F. ANEXARTITOS	1

We therefore observe that in this case the potential political coalitions to form a government are the {AB}, {AC}, {BC}, since it wouldn't be desirable to have a large number of parties in government coalition. Similarly to previous match, we could predict that the combination of {AC} is the most likely form coalition with {BC} as the second most likely, since the party {A} has accumulated the largest number of seats and cooperates with the party {C}, which has the minimum number of seats due to government coalition.

TABLE 4

Coalition and number of seats

Minimum number of parties:	AB	AC	BC
Number of seats:	276	170	160

Before that, for the first time in the modern history of Greek parliament were elections after exactly four years on June 18, 1989. The ND was the first party and PASOK second, but there was not absolute majority for anyone. After negotiations they formed a coalition between ND and Synaspismos under Mr. Tzanetakis. The table below shows the number of seats each political party based on official data.

TABLE 5

Party and number of seats

Party	Number of seats
A. ND	145
B. PASOK	125
C. SYNASPISMOS	28
D. DIANA	1
E. EMPISTOSYNI	1

The necessary number of seats in order to have a government coalition is at least 151 seats. We therefore observe that potential political coalitions to form a government are the {AB}, {AC}, {BC}, since, as we already mentioned, it would not be desirable to have a large number of parties in a government coalition. In addition, the parties wish to cooperate with parties that have received as little as possible votes, thus maximizing their share of the coalition. In the following table, we could predict that the combination of {AC} is the most likely form coalition with {BC} as the second most likely.

TABLE 6

Coalition and number of seats

Minimum number of parties:	AB	AC	BC
Number of seats:	270	173	153

Studying these two elections we are ready to say that they have too many things in common, since only three parties received a sufficient number of seats, while the main party in seats in both contests highlighted the same party. Using the data of the first election and with help from the model of political ideology, place the parties in one dimensional continuous chart from left to right, in relation to their political ideologies. Basis of the data of the chart (Converse and Valen, 1971), and matching in our example parties' election in June 1989 instead of parties in Norway shows the following graph:

TABLE 7

Liberal vs Conservative

	C		B	E	D	A	
Liberal	-5	0	4	6	7	11	Conservative

In this example, {BC} and {ABDE} are the minimum possible coalition's victory. The combination of {AC}, for example, is not connected.

Many times the political strategies of parties not completely coincide with their political ideologies, so a party that is liberal on social issues, may follow conservative policies on other issues. As we have already said Converse and Valen (Converse and Valen, 1971) use a corresponding graph having as basis the political parties in Norway for cultural affairs of the country. Using the data in the chart and placing the Greek parties' first election where the political culture we have:

TABLE 8

Liberal vs Conservative

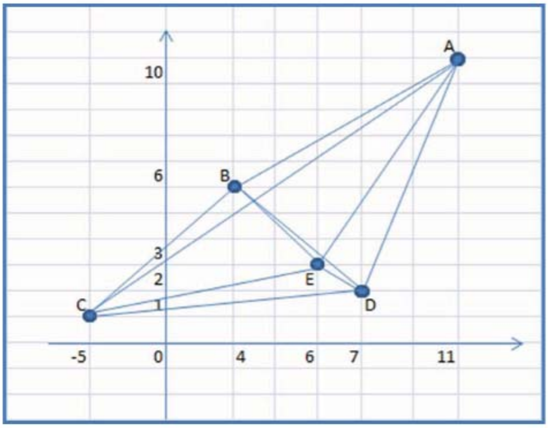
	C	D	E	B	A	
Liberal	1	2	3	6	10	Conservative

In this case the coalition {AB} is a minimal winning coalition. Using a two-dimensional Cartesian system will try to calculate the relative ideological closeness of the parties (Fig. 1).

What we observe is that party {A} and {C} while forming coalition governments in two consecutive matches appears diametrically opposite to our chart. Although each party wants to be part of a coalition that adopts a platform near the ideological point of his party, this seemed not to apply to the 1989 elections because the coalition parties came from ideologically opposed political orientations. Perhaps this was to be one of the reasons that two continuous coalitions had a very short lifetime.

FIGURE 1

Two-dimensional spatial placement of Greek parties



3.2.2 January 2013

According to the last voting intension survey the status of potentially voting results gives the following data (Metron Analysis, 2013). Same results for the potentially voting percentage between ND and SYRIZA. We will try to show the possible combinations of political coalitions to form the government. The first table shows the number of seats each political party based on survey data (Metron Analysis, 2013).

TABLE 9

Party and number of seats

	Party	Intention of voting	Number of seats
A.	SYRIZA	18.8%	126
B.	ND	18.7%	76
C.	XRISI AYGI	7.2%	29
D.	PASOK	5.2%	21
E.	AN. ELLINES	4.3%	17
F.	DIM. ARISTERA	3.8%	15
G.	KKE	3.6%	15
H.	OIK.PRASINOI	1.2%	-
I.	ALLO	4.4%	-
J.	AKYRO - LEYKO	10.2%	-
K.	DEN PSIFIZO	14.8%	-
L.	NO ANWSER	7.8%	-

Watching the poll percentages easily conclude that no party can have the required number of seats to form government. Aware also of the political positions and attitudes of the parties, we conclude that the minimum number required in order to form a coalition government, are three parties.

TABLE 10

Coalition and number of seats

Minimum number of parties:	AEF	AEG	AFG
Number of seats:	159	159	156

The Left parties collectively gather the largest number of seats, thus creating the conditions for forming a government. Unlike the ND with the low number of seats they have held more than four parties to form a government, except SYRIZA, which is theoretically impossible. Knowing the positions of the parties on the possibility of cooperation with other parties see that five of the seven parties that appear to enter the parliament is willing to cooperate in government. So the available parties could cooperate is finally five. We conclude that the only combination that can lead to a coalition government is the {AEF} with 159 seats. Using the model of political ideology (Converse and Valen, 1971), we place these five parties in one-dimensional continuous chart from left to right, in relation to their political ideologies.

TABLE 11

Liberal vs Conservative

	F		A	E	D	B	
Liberal	-5	0	4	6	7	11	Conservative

To be able to arrive at a graph that will reflect the relative ideological closeness of the parties will present the five parties according to their policies on issues of the economy (Converse and Valen, 1971).

TABLE 12

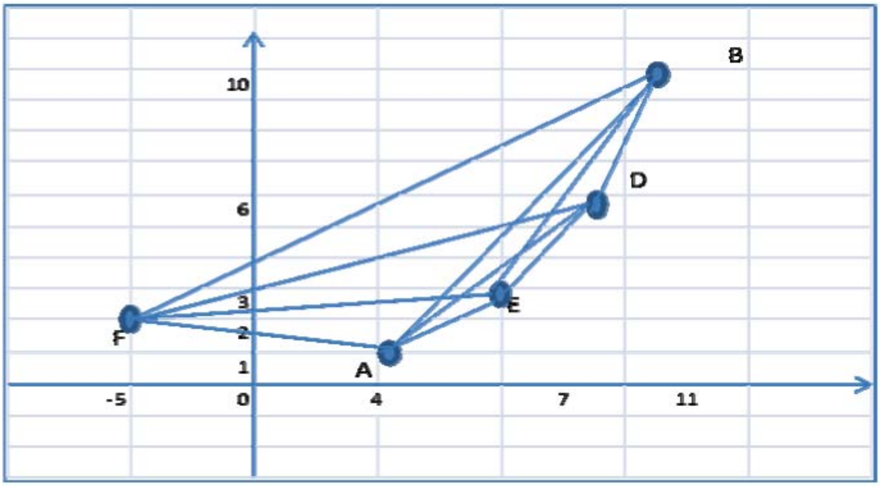
Liberal vs Conservative

	A	F	E	D	B	
Liberal	1	2	3	6	10	Conservative

Using again a two-dimensional Cartesian system will try to calculate the relative ideological closeness of the parties (Fig. 2).

FIGURE 2

Two-dimensional spatial placement of Greek parties



We therefore observe that the coalition {AEF} presented clumpier than other coalitions and that can lead to the formation of coalition government.

4. Conclusion

According to Shapley (1979), the theory of games might be called the mathematics of competition and cooperation. Comparing gaming and game theory we can see two extremely different yet highly intertwined disciplines (Shubik, 1971). Analyzing those cases where the choice of one party government is not possible we can potentially suggest which parties are most directly tied to coalition structure. However, in a game where the parties are characterized by their ideology, the negotiated rate is derived by each party in government. At the same time the parties negotiate the policies that the coalition government will follow. In our example we used methods that can be the driver of cooperation. A variety of results can set the working frame of decision, bargain, conflict and strategy.

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