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MODELING UNCERTAINTY AND RISK IN INVESTMENT APPRAISAL DECISIONS: A MONTE CARLO SIMULATION APPROACH

By

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Abstract

The present paper deals with the implementation of risk analysis in engineering economic problems, using Monte Carlo simulation. The paper consists of two parts. After a short introduction about risk analysis methods used in engineering economic problems, the first part discusses the role of simulation, especially Monte Carlo simulation, in risk analysis and presents an extended review of studies, using either statistical techniques in general, or specifically Monte Carlo simulation in risk analysis. In the second part of the paper, we construct a model of a Monte Carlo simulation for the appraisal of a potential investment with uncertain annual revenues and costs, using Excel Spreadsheets and Visual Basic. The implementation and use of the model is demonstrated with a numerical example. The results obtained show that Monte Carlo simulation can prove a valuable technique in the decision making for the evaluation of a potential investment. JEL Classifications: CIS, Gl 1, M10.

Keywords: investment appraisal, Monte Carlo simulation, decision making.

1. Introduction

While management science is developing through out the years, day-to-day managers' tasks become more and more complicated. Decision making to undertake a potential investment is one of the basic fields in the management science and especially in the top-level management. This exactly is the object of a relatively new economic field, engineering economy; to enable managers and engineers to evaluate in accuracy the economic consequences of major capital investments. An investment project's impact on the profitability of an organization is often significant; therefore the evaluation of the financial risks of the potential investment is more than essential before making the final decision to undertake it. Although some of the input information in an engineering economic problem (such as the cost of equipment or the current tax rate) can be well defined and their quantities are deterministic, most of the required information is uncertain, such as the cash flows from revenues and costs, the interest rates, the project life or others such as the cost of labour and raw materials, the level of demand etc. In the case of an investment appraisal decision, the usual non-deterministic variables are the revenues and costs generated from the potential investment (Goyal *et al.*, 1997).

Capital investment is about identifying, analyzing and managing uncertainty. Additionally, risk is a part of any investment decision. Assessing uncertainty and risk is an important and often complex task in reaching effective capital investment decisions. The term risk focuses on the potential gain or loss (of economic or other nature) resulting from the investment decision. Projects with low probability (or small magnitude) of loss may be judged to have a relatively low risk, while on the other hand, projects with high probability (or high magnitude) of loss may be judged too risky for implementation.

Generally, risk handling methods fall into two broad categories: simple risk-adjustment methods and risk analysis. Risk-adjustment methods are mainly based on deterministic estimations and proper adjustments to the Discounted Cash Flows model but despite the fact that they are easy to use, they contain assumptions that may not be clearly understood. On the other hand, risk analysis techniques emphasize the awareness of the uncertainties that influence critical project variables. The increased risk information they offer not only improves the understanding of the nature of risks and reduce forecasting errors, but provides managers with many qualitative benefits as well (Ho and Pike, 1998).

The importance of risk analysis in the evaluation of capital investments has been pointed out by many studies. Pike (1988; 1989) conducted a survey based on a sample of 100 large UK firms, examining the capital budgeting practices in investment selection techniques over an 11-year period, in years 1975, 1980 and 1986. The purpose of the study was to examine the trend towards greater sophistication in investment selection techniques and their resulting impact on the effectiveness in evaluating major capital projects. From the analysis of responses by the sample firms, Pike concluded that there had been remarkable increases in the use of sophisticated investment selection techniques, particularly in risk analysis (from 26% in 1975 to 86% in 1986) with the use of discounted cash flow and sensitivity analysis techniques. According to Pike, this increase was closely associated with higher levels of capital investment effectiveness, as perceived by the firms' senior managers.

Scenario analysis is one of the most common approaches in risk analysis, incorporating worst- and best-case scenarios. However, these scenarios are neither easy to be interpreted and do not provide probabilities of occurrence, nor they provide other useful information such as the probability of money loss invested in a project. Another approach of handling economic risk is sensitivity analysis, a useful technique that can provide insight into risk analysis problems, although it is inappropriate when statistical dependence exists between variables (Eschenbach and Gimpel, 1900). Other commonly used risk analysis techniques for large-scale project evaluation include probability analysis, decision-tree analysis or Monte Carlo simulation, a technique which combines risk analysis and simulation and will be discussed in the next paragraph (Ho and Pike, 1998).

2. The role of simulation in handling economic risk

The term *simulation* is used in reference to any analytical method that imitates a real-life system, especially when other analyses are too mathematically complicated or too difficult to be reproduced. A *simulation model* is a system that can be "taught" how the real system would react in various conditions. Most of these models are constructed as a series of mathematical equations. The simulation experiment involves the interchange of a number of input variables, in order to determine the impact of their various combinations on one or more output variables. *Computer simulation models* are systems using the aid of computer hardware and software and most of the times have the form of an electronic spreadsheet. Such models are implemented in order to demonstrate how spreadsheets can be used to describe the philosophy of a simulation model, to perform the necessary calculations, to generate the simulation data and provide a summary of the simulation results.

While spreadsheets may appear to be valuable in conducting single simulation studies, they are generally limited to smaller and less complicated simulation models, since they can only reveal a single outcome, generally the most likely or average scenario. As a system grows in complexity, other computer procedures may be necessary to adequately model it and carry out the simulation calculations. Spreadsheet risk analysis uses both a spreadsheet model and a simulation to automatically analyze the effect of various input variables on one or more output variables of the modeled system. General-purpose computer programming languages, such as Visual Basic, can be used to develop a computer program that will model the system and perform the simulation computation. The advantage of using general-purpose programming languages rather than a single spreadsheet is that such languages have greater flexibility to model more complex systems. Based on the development of simulation applications, both users of simulation and developers of computer software realized that computer simulations have many common features, including generating values from probability distributions, maintaining a record of what happens during the simulation process, recording simulation data and finally summarizing the simulation results (Seitz and Ellison, 1999).

The *Monte Carlo method,* as it is known today, encompasses any statistical sampling technique employed to approximate solutions to quantitative problems. The first researcher who worked on this method was Ulam. His contribution was to recognize the potential of the newly invented electronic computer to automate such sampling. Ulam did not invent statistical sampling. It had been used to solve quantitative problems before, employing natural processes (such as dice tosses or card draws) in order to generate samples. Working with John von Neuman and Nicholas Metropolis, he developed algorithms for computer implementations and explored means of transforming non-random problems into random forms that would facilitate their solution via statistical sampling. This work transformed statistical sampling from a mathematical curiosity to a formal methodology, applicable to a wide variety of problems. It was Metropolis who named the new methodology after the casinos of Monte Carlo. Metropolis and Ulam published the first paper on the Monte Carlo method in 1949[2] (Seitz and Ellison, 1999).

The Monte Carlo technique, coupled with a simulation model, yields a simulation technique called Monte Carlo simulation. This technique randomly generates values for uncertain variables, over and over again, in order to simulate a model and is used for over two decades in capital investment analysis. Supported by appropriate software, computer spreadsheets (such as Microsoft Excel) can be easily transformed into Monte Carlo simulation models. Analytical results can be reached by employing suitable application software (such as Mathematica)[3] to simplify the necessary calculations (Goyal *et al.,* 1997).

3. Literature review

Capital investment evaluation methods are distinguished in the international literature between “naïve” and “sophisticated” (Pike, 1988). The former include mainly simple financial appraisal techniques, like payback or accounting rate of return, while the latter include scientific financial appraisal techniques, like discounted cash flow methods (of which the internal rate of return and net present value methods are the best known), risk analysis techniques, like sensitivity analysis, probability analysis, scenario analysis etc., or management science techniques, like mathematical programming, computer simulation, decision theory etc. The next two paragraphs present and comment on some popular studies in investment evaluation under uncertainty. These studies are presented in two separate groups, with the second group focusing to the studies using the Monte Carlo simulation method.

3.1 Studies using statistical techniques in risk analysis of capital investment projects

Hillier (1963) was the first one who proposed the use of probability distribution of the present worth in project risk analyses. He showed that this probability distribution of the measure of the present worth (usually the Net Present Value, NPV) is normal and can be derived, under certain assumptions, from annual cash flows that are themselves random variables. He also presented equations for the NPV parameters (the mean value and the standard deviation) when the cash flows were mutually independent random variables and when the cash flows were perfectly correlated. Giaccotto (1984) tried to span the gap that existed in the literature when dealing with non-perfectly correlated cash flows in the context of capital budgeting risk-return analysis. Based on the work of Hillier, he introduced a new methodology that allows dependence in a project's cash flows and found that the serial correlation of cash flows may affect the expected NPV of a project.

Eschenbach and Gimpel (1990) dealt with the uncertainty in engineering economic problems presenting a variation of traditional sensitivity analysis, called stochastic sensitivity analysis, a method with wide applications in mathematical modeling as well. This analysis includes probability data about the variables and isolates the effects and the relative importance of individual variables. The variables' probability distributions are the input, as in simulation, while the output relates changing variables to changes in present worth (which is the model's outcome), as in deterministic sensitivity analysis.

While simulation focuses on the present worth's cumulative distribution function, Eschenbach and Gimpel while presenting stochastic sensitivity analysis, described a probability metric that can be used to connect uncertainty in individual variables with uncertainty in present worth. The present worth for each variable was graphed against the variable's cumulative distribution function. They defined the expected value of present information (EVPI) as the difference between the expected monetary value (EMV) of a decision when perfect information about the future state of nature is known, and the EMV with the information available at the time that the decision is made. They calculated the present worth's conditional expected value and the EVPI for each variable, as well as the probability of breakeven. Consequently, the relative importance of each variable could be evaluated and compared, since all variables were graphed on the same x-axis.

Eschenbach and Gimpel demonstrated the application of stochastic sensitivity analysis with an example of an investment decision model for an electric power utility, which evaluates the construction of a hydroelectric dam to meet future growth in power demand by either hydroelectric power or gas-fired turbine generation. The variables used in the model were dam construction cost, cost of gas turbine generation, discount rate and power demand growth rate.

3.2 Studies using Monte Carlo simulation in engineering economy problems

The implementation of Monte Carlo simulation in financial and investment analysis has been reported for over two decades. In the early 80's, Coats and Chesser (1982) used Monte Carlo techniques along with classical financial statement analyses in order to produce useful statistical measures, such as probabilities of occurrence, confidence intervals and standard deviations, in addition to standard financial reports. Later on, Seila and Banks (1990) simulated financial risk with Monte Carlo techniques, by exploring the probability distribution of the NPV of a project as a function of the unknown random input variables. As the performance measure of the model, that is the NPV, is also a random variable they tried to evaluate risk associated with decisions based on it, due to the uncertainty in its value. They applied Monte Carlo simulation in an electronic spreadsheet and illustrated the whole procedure with an example, using formulas to generate random values with the aid of LOTUS 1-2-3.

Alloway (1994) supported the applicability of electronic spreadsheets in engineering economic analysis. Classifying the related software into three categories; pre-written application software, custom-written software and productivity tools (a form which includes electronic spreadsheets), he characterized spreadsheets as *"a hybrid between the two ends of the software spectrum"* which incorporate the advantages of each approach while avoiding their disadvantages. Comparing them with the other two approaches or with closed form solutions (like the expressions given in text books) he concluded that, electronic spreadsheets currently provide the greatest benefit/cost ratio. He also argued that spreadsheet advantages include wide applicability to almost every subject, low demands on time or training, low cost and minimum demands on solution or presentation time.

The modeling capability of a spreadsheet in complex economic engineering problems was demonstrated by Alloway with a Monte Carlo simulation example, where the objective was to determine the expected present worth for an alternative investment when several cash flows were uncertain. The spreadsheet model consisted of four regions: i) the input area that showed the distribution and the parameter values for each cash flow type, ii) the simulation area that determined the present worth for each trial, iii) the summary information area that provided statistics used by the modeler to reach a decision and iv) the random number generator area that provided the values used in the simulation section.

The present worth of the project was determined as a function of variable elements, including life, salvage, annual savings and expenses, whose values were based on the random numbers generated in the fourth region of the model. Each random cash flow was modeled separately and entered into a single column. Each row in the spreadsheet represented one of the total 1.000 trials executed in the simulation, modeled by using different versions of the Lotus 1-2-3 software. The results of the simulation were evaluated graphically using a bar chart of the present worth for the 1.000 trials, since such kind of charts gives a better impression of the present worth's distribution than the summary statistics. Additionally, the cumulative average present worth was plotted to determine if the simulation had reached steady-state with the 1.000 trials.

Apart from using later versions of the Lotus 1-2-3 software, Alloway also experimented with the use of add-in software, such as @RISK, in order to simplify the initial simulation model. Comparing three different Lotus 1-2-3 models (Release 1.1, Release 4 and @RISK with DOS version 2.2) for 20 simulations of 1.000 trials each, he found no significant difference in the average present worth values.

Coates and Kuhl (2003), in a more recent paper, provided three simple examples demonstrating the ease with which engineering economy problems with stochastic input variables can be modeled using widely available industrial simulation software. In the examples they presented, the probability descriptions of the random input variables, along with Monte Carlo techniques, provided a practical method of finding the distribution of the desired output variables, using simulation packages that can handle great amount of sampling data and have capabilities of good output report.

In their first example, they demonstrated the calculation of the future worth of an annual series of payments, represented by the NPV, where the interest rate varies from year to year. They assumed a stock market investment for the entire time period of the payments, with a stable long-term average return but individual annually returns normally distributed with a given standard deviation. The interest rates were selected via Monte Carlo sampling from the distributions. In the case of a fixed and known interest rate, the calculation of the future worth would be straightforward, through the classical NPV formula. They used the simulation software SLAM II instead, to estimate the NPV distribution for a great number of repetitions. From the reported summary statistics the range of the future worth was determined, while the standard future worth formula would only give a point estimate with no indication of the probable range.

The second example of their paper attempted to model the risk in the appraisal of an investment project, having uncertain, mutually independent, normally distributed, annual cash flows, as in Hillier's (1963) initial example. Additionally, to make the problem more complicated, they allowed the interest rate to vary from year to year. An initial random starting value was assigned to the project's interest rate of the first year. The rate of each subsequent year was generated by a first order autoregressive stochastic process, as in Giaccotto (1984). Moreover, the project life could vary from 4 to 6 years, with a given chance for each one of the three scenarios (4, 5 or 6 years). As shown in their example, the importance of including the variability of interest rates and project life in an investment appraisal problem was indicated by the fact that the probability of a negative NPV, in such a case, could rise substantially (even up to 10 times) than in a similar problem with uncertain yearly cash flows only. Moreover, even thought the mean NPV in both cases could be similar, or even the same, the standard deviation of the NPV distribution might double.

In their final example, Coates and Kuhl compared two mutually exclusive, alternative projects with different net expected cash flows, normal cash flow distributions and interest rate distributions like the one described in their second example. The comparison between the two projects was based on the difference in the expected Net Present Values of their investments. They applied a simulation model on each alternative project, as the one described in the previous paragraph, obtained independent observations of the NPV for each one and as a result they constructed a confidence interval of the difference between the population means. Using common random numbers, they treated the corresponding independent observations of the NPV from each project as matched pairs while constructing the confidence interval. After that, a point estimate of the mean difference in the NPVs of the two alternatives was calculated and a confidence interval of this mean difference was constructed. From the sign of the mean difference and the range of the confidence interval they concluded on which alternative would yield a higher return. Coates and Kuhl argued that since in most investment problems the decision maker will only have one opportunity to invest in any particular project, therefore a better analysis technique would be to construct a tolerance interval on the NPV difference for a single investment instead.

Perry (2006) presented an overview of the Design for Six Sigma process (a methodology that spans the entire product commercialization process from business idea development to initial product sales), utilizing specific applications of Monte Carlo simulation using Crystal Ball® software. Among others, he demonstrated how Monte Carlo simulation along with product optimization techniques could be applied in business financial value analysis. Doing so, he presented a case study example of a new product design project. In his example a final financial analysis was necessary, at the last phase of the project, since an initial one, during the early stages, preceded. Once the primary variables of the initial financial analysis (sales volume, unit price, raw material unit cost, operating/other cost per unit etc) and their distribution assumptions (distribution type, mean value or standard deviation) were defined, a traditional financial analysis was carried out in order to determine the expected value of the project's NPV. Having executed a Monte Carlo simulation and according to the distribution type of the estimated values, it was obvious that, although the project was expected to produce a positive NPV, it was not statistically certain. In this example, the simulation results indicated that there was a 20% chance of a negative NPV of the project, a possibility that should be taken under serious consideration despite of the positive expected NPV.

4. An application of Monte Carlo simulation: Investment appraisal

4.1 The Net Present Value (NPV) Method in investment appraisal

The most common scientific method for investment evaluation is the Net Present Value (NPV) Method. In this paper we construct a Monte Carlo simulation of the appraisal of a potential investment based on the investment's NPV, using Excel spreadsheets and Visual Basic. In a potential investment project with uncertain annual revenues and costs, using the above software tools, we can randomly reproduce their values, estimate the annual cash flows and finally calculate its NPV, not just once but for a great number of trials. Doing so, we have the opportunity to statistically evaluate the results using a number of statistical modules provided by the Excel spreadsheets (Anderson *et al,* 1997). A sufficient number of repetitions should be executed, in order for the simulation to provide reliable results in the appraisal of the potential investment. The fact that the revenues generated and the costs raised are not historical data but estimates of unknown quantities, should be taken seriously into account while applying the probability of occurrence to each value (Marsaglia and Zaman, 1991).

4.2 Methodology

Variable Definition

First, we have to set the variables that will be used in our simulation. These variables are

- **Revenues** **:** the annual generated revenues from the investment
- **Costs** : the annual out coming costs of the investment

- **Years** : the number of years that the investment lasts
- **Random (Rn) :** the random numbers used in the simulation
- **Times** : the number of trials executed by the simulation

Construction of the Simulation and Visual Basic modules used

In order to construct the simulation required for the appraisal of the potential investment, we use the Visual Basic language. Visual Basic is a tool, provided by the Excel spreadsheets, that enables us to use simple commands to program the simulation. For the Revenues generated from the investment, we use the module: *«Dim Revenue, Years, Rn, Times»,* setting the variables that will be used in our simulation. Having the variables defined, we set the years of the investment's life. If, for example, we want to set five years, the module that we have to use is the following: *«For Years = 2 to 6».* The numbers 2 to 6 correspond to the columns of the Excel spreadsheet that will be used to place the values for each one of the five years. The next step is to decide how many trials we wish the simulation to execute. In order to do that, lets say for 201 trials, we use the module: *«For Times = 15 to 215».* This time the numbers 15 to 215 correspond to the rows that the Excel spreadsheet will use to place the 201 trials of the simulation.

After that, we have to command the simulation to select randomly a value for each years' Revenues according to a given range of probabilities. To do that we first create a box in the Excel spreadsheet listing all the probabilities (both separately and cumulatively) and the values of the Revenues which correspond to each probability. The module in the Visual Basic is: *«If Rn<Range("X") Then Revenue = Range("Y")»,* where "X" is the cell with the cumulative probability and "Y" is the cell with the Revenues' value corresponding to the probability. The factor "Range" is used in order to make the simulation user - friendly and enable a manager, changing the probabilities and the Revenues' values, to run the simulation for different investment options (Castillo-Ramirez, 2000).

The modules' list used to create the simulation for Revenues is the following (Deitel et al, 1999)

```
Sub Nikolaos()
Dim Revenue, Years, Rn, Times
For Years = 2 To 6
For Times = 15 To 215
Rn = Rnd()
If Rn < Range("D3") Then Revenue = Range("E3")
If Rn >= Range("D3") And Rn < Range("D4") Then Revenue =
Range("E4")
```

```
If Rn > = Range(“D4”) And Rn < Range(“D5”) Then Revenue =
Range(“E5”)
If Rn >= Range(“D5”) And Rn < Range(“D6”) Then Revenue =
Range(“E6”)
Cells(Times, Years) = Revenue
Next
Next
Nikolaos 1
End Sub
```

Exactly the same way we construct the simulation for the Costs that the investment will arise. The last module used in Visual Basic enables the user of the simulation to exit the program by just pushing one button. This module is (Deitel *et al.,* 1999)

```
Sub Exitprog ()
Active Workbook. Close
End Sub
```

4.3 Analysis

Running the Simulation

After the construction of the simulation we can proceed to the analysis. First, we estimate each year's Cash Flows, according to the values of Revenues and Costs chosen randomly from the simulation abstracting Costs from Revenues. Estimating the Cash Flows for each one of the five years, we can very easily calculate the investment's NPV using the respective formula provided by the Excel spreadsheets. A useful function of the simulation is the *«Run Simulation Revenue/Cost»* button which enables the user to run the simulation automatically.

Statistical Modules

Having the simulation executed, we can evaluate various statistical modules very useful in the interpretation of the results. The statistical figures evaluated are the *Minimin, Maximax, Median, Mean* and *Standard Deviation.* The population used for the calculation of these figures is the NPVs. **Minimin** is the minimum value of the population while **Maximax** is the maximum. The **Median** is defined as the score in a population that divides it into two equal parts; it is the middle score of a set of scores ranged in ascending order. The **Mean** is the measure of the central tendency that most of us recall when we hear the term «average». It is simply the arithmetic average of a distribution of scores. If the Mean and the Median in a population have similar values, then the distribution of the population approximates the normal one. Finally, the **Standard Deviation** is a measure of dispersion of the distribution and describes the typical or average deviation of the scores around their Mean (Walsh, 1990).

5. A numerical example

In the last section of the paper, a hypothetical example will be presented to introduce a simulation model as a valuable technique in the appraisal of an investment project whose cash flows are non-deterministic variables. In the process we demonstrate how risk analysis provides the basis for the decision to accept or reject a potential investment.

5.1 Data

Supposing a 5-year investment project with an initial outflow of €40.000,00 at the beginning of the 5-year period. We assume that for each one of the 5 years, the revenues generated by the investment may vary from €40.000,00 to €60.000,00, following a distribution approximating the normal one. Transforming this fact to probabilities, we choose four discrete values of revenues and apply a probability of occurrence to each value, as shown to the list bellow. The same way, each year's costs may vary from €25.000,00 to €40.000,00 following again a normal distribution; consequently, we consider four different values of costs as well as their probability of occurrence, which are presented in the same list.

Year	*Outflow (€)*	*Revenues (€)*	*Probability*	*Costs (€)*	*Probability*
0	40.000	-	-	-	-
1 to 5		40.000	15%	25.000	10%
	-	50.000	40%	30.000	25%
		55.000	30%	35.000	35%
		60.000	15%	40.000	30%

The final step is to set the NPV rate. We expect from the investment project an at least 12% rate of return, therefore the NPV rate in the calculations for the estimation of the investment's NPV will be 0,12.

5.2 Analyses and interpretation of results

Having all the data of the example defined, we can proceed to the simulation. In this example we set the simulation to execute 200 trials, meaning to select from the previous list, for each one of the 5 years of the investment, 200 values for revenues and costs, in random order. The first 15 values of revenues and costs, for the 5 years respectively, are presented in the following Table 1:

TABLE 1

SIMULATION of "REVENUES" and "COSTS"

200-Times	*REVENUE* Year 1	Year 2	Year 3	Year 4	Year 5	*COST* Year 1	Year 2	Year 3	Year 4	Year 5
1st	55000	50000	50000	50000	50000	40000	25000	30000	40000	30000
2nd	50000	55000	50000	50000	55000	35000	40000	35000	40000	25000
3rd	55000	50000	55000	55000	55000	40000	40000	35000	40000	30000
4th	50000	50000	40000	50000	50000	30000	25000	30000	40000	35000
5th	50000	50000	50000	50000	55000	25000	35000	40000	35000	40000
6th	55000	55000	55000	55000	55000	40000	40000	40000	40000	35000
7th	40000	50000	50000	55000	60000	35000	30000	35000	35000	40000
8th	55000	50000	50000	40000	60000	40000	35000	25000	35000	40000
9th	55000	50000	60000	40000	50000	40000	35000	35000	40000	35000
10th	55000	50000	50000	55000	55000	40000	40000	25000	40000	40000
11th	40000	55000	40000	55000	55000	40000	30000	35000	35000	40000
12th	50000	50000	50000	55000	50000	25000	40000	40000	35000	30000
13th	60000	55000	50000	50000	55000	40000	35000	35000	30000	30000
14th	55000	60000	55000	50000	50000	30000	30000	25000	35000	35000
15th	50000	50000	60000	50000	50000	30000	25000	35000	30000	40000

According to these values, the simulation calculates each year's cash flow and the investment's NPV, for 200 trials. Again, the first 15 results are presented in the following Table 2.

TABLE 2

ESTIMATION of "CASH FLOWS" and "NET PRESENT VALUE"

CASHFLOW					
Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**NPV**
15000	25000	20000	10000	20000	£25.262,03
15000	15000	15000	10000	30000	£19.405,46
15000	10000	20000	15000	25000	£19.318,84
20000	25000	10000	10000	15000	£19.771,38
25000	15000	10000	15000	15000	£19.441,31
15000	15000	15000	15000	20000	£16.908,78
5000	20000	15000	20000	20000	£15.143,77
15000	15000	25000	5000	20000	£17.671,40
15000	15000	25000	0	15000	£11.656,67
15000	10000	25000	15000	15000	£17.203,48
0	25000	5000	20000	15000	£4.710,51
25000	10000	10000	20000	20000	£21.470,07
20000	20000	15000	20000	25000	£31.373,76
25000	30000	30000	15000	15000	£45.634,83
20000	25000	25000	20000	10000	£33.966,13

Since the 200 NPVs have been calculated we perform the statistical analysis of the results estimating the Mean, the Median, the Standard Deviation and various characteristic Tolerance Intervals of the NPV population. The overall results of the analysis are reported in Table 3.

TABLE 3

OVERALL RESULTS

REVENUE								**COST**		
Probability	Cumulative	Cashflow €			NPV rate			Probability	Cumulative	Cashflow €
0,15	0,15	40.000 €			0,12			0.10	0.10	25 000 €
0,40	0,55	50.000 €						0,25	0,35	30 000 €
0,30	0,85	55.000 €	Maximax	Minimin	Median	St Dev	Mean	0,35	0,70	35.000 €
0,15	1,00	60 000 €	52.519,57 €	-2793,01 €	22.338,28	11995,79	22 117,77 €	0,30	1,00	40.000 €

According to these results, the Mean of the NPV population is €22.177,77 and the Median €22.338.28, a fact that proves that our population follows the Normal Distribution. Since the distribution is normal we can easily perform a statistical analysis of the results, using the Standard Deviation. In a Normal Distribution, exactly 68,26% of the total area of the values falls between ±one Standard Deviation from the Mean. Since in our case the Standard Deviation is €11.995,79 (see Table 3) the range of the NPV will be €10.121.98 to €34.113,56, with approximately 68% accuracy. In other words, 68 times out of 100, the NPV of the investment will be greater than €10.121,98 and smaller than €34.113,56. Furthermore, exactly 95,44% of the total area of the values falls between ±two Standard Deviations from the Mean, so in our example the range of the NPV of the investment will be €-1.873,81 to €46.109,35 in 95% of the cases. This result states that if we want to broaden the tolerance interval to 95%, there is indeed a slight chance the investment's NPV to be negative, so the investment should be rejected. However, if we accept a little smaller tolerance interval, the NPV will take only positive, or close to zero, scores so we can conclude that in approximately 90% of the cases the investment should be undertaken.

5.3 Cumulative Results and Conclusions

Obviously, every time we run the simulation slightly different results will come out because the random values of revenues and costs will differ. Nevertheless, these results appear to be, more or less, the same with those shown in Table 3. The cumulative results for four more runs (together with the ones of the first case), both the 68% and the 95% tolerance interval included, are summed in Table 4.

TABLE 4

CUMULATIVE RESULTS

<table>
<tr><th>Statistics</th><th>Results</th><th colspan="2" rowspan="4">Variance with 1 standard deviation (68% accuracy)</th><th colspan="2" rowspan="4">Variance with 2 standard deviation (95% accuracy)</th><td rowspan="6">First Run</td></tr>
<tr><td>Maximax</td><td>52.519,57 €</td></tr>
<tr><td>Minimin</td><td>-2.793,01 €</td></tr>
<tr><td>Median</td><td>22.338,28 €</td></tr>
<tr><td>StDev</td><td>11.995,79 €</td><td rowspan="2">34.173,56 €</td><td rowspan="2">10.181,98 €</td><td rowspan="2">-1.813,81 €</td><td rowspan="2">46.169,35 €</td></tr>
<tr><td>Mean</td><td>22.177,77 €</td></tr>
</table>

<table>
<tr><th>Statistics</th><th>Results</th><th colspan="2" rowspan="4">Variance with 1 standard deviation (68% accuracy)</th><th colspan="2" rowspan="4">Variance with 2 standard deviation (95% accuracy)</th><td rowspan="6">Second Run</td></tr>
<tr><td>Maximax</td><td>63.669,10 €</td></tr>
<tr><td>Minimin</td><td>-1.984,63 €</td></tr>
<tr><td>Median</td><td>24.026,58 €</td></tr>
<tr><td>StDev</td><td>12.732,33 €</td><td rowspan="2">36.731,63 €</td><td rowspan="2">11.266,97 €</td><td rowspan="2">-1.465,36 €</td><td rowspan="2">49.463,96 €</td></tr>
<tr><td>Mean</td><td>23.999,30 €</td></tr>
</table>

Statistics	Results	Variance with 1 standard deviation (68% accuracy)		Variance with 2 standard deviation (95% accuracy)		
Maximax	52.051,15 €					
Minimin	-13.643,43 €					Third Run
Median	22.271,96 €					
StDev	**12.345,34 €**	34.204,39 €	9.513,71 €	-2.831,63 €	46.549,73 €	
Mean	**21.859,05 €**					

Statistics	Results	Variance with 1 standard deviation (68% accuracy)		Variance with 2 standard deviation (95% accuracy)		
Maximax	60.501,41 €					
Minimin	-17.547,69 €					Fourth Run
Median	22.358,57 €					
StDev	**12.633,70 €**	35.168,98 €	9.901,58 €	-2.732,12 €	47.802,68 €	
Mean	**22.535,28 €**					

Statistics	Results	Variance with 1 standard deviation (68% accuracy)		Variance with 2 standard deviation (95% accuracy)		
Maximax	56.037,12 €					
Minimin	-22.794,89 €					Fifth Run
Median	21.373,06 €					
StDev	**12.471,83 €**	33.179,98 €	8.236,32 €	-4.235,51 €	45.651,81 €	
Mean	**20.708,15 €**					

According to these values, in the 68% tolerance interval, the NPV takes only positive values, in all cases. However, the results show that when the tolerance interval increases to 95%, the NPV can take slightly negative values, with a minimum value of €-4.235,51, in the fifth run. Taking into account the results from all five runs, we can conclude that the investment in question is worthwhile to be undertaken since, with approximately 90% accuracy, the investment will return positive Net Present Value.

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Notes

1. An earlier version of the paper was presented in the 6th Annual International Conference of the European Economic and Finance Society (EEFS), 31st May - 3rd June 2007, Sofia, Bulgaria, under the title: “Construction of a Spreadsheet Model using Monte Carlo Simulation for the Appraisal of a Potential Investment”

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