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CORPORATE CONTROL AND INVESTMENT DECISIONS UNDER UNCERTAINTY WITH INCOMPLETE MARKETS

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Abstract

In a stock-market economy under certainty, or uncertainty but with a complete set of asset markets, the objective of the firm is profit maximization. In an economy under uncertainty with incomplete asset markets, profit maximization is not a well-defined objective. We present a general equilibrium model of firm's investment decision under uncertainty based on the preferences of major shareholders and their corporate control power over production outcomes and we compare it to some well-established investment decision criteria.

Keywords: *firm behaviour, incomplete markets, Shapley value.* JEL: D210, D810, D520, C710.

1. Introduction

In general equilibrium theory, the firm is viewed as an entity whose objective is to utilize efficiently its technology in order to maximize profit. The level of production of a firm depends on the nature of its technology, a factor usually considered as exogenous, and on the price of the commodity produced. Assuming that the economy is competitive, given the technology and equilibrium market prices, the firm has enough information in order to qualify a production choice as optimal.

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Whether the ownership structure of the firm consists of a single owner or a group of shareholders is irrelevant to the objective of the firm since the latter depends only on technology and prices, parameters that are considered as given and are out of the control of a competitive firm. Even if one insisted that the production decision of a firm should depend in one way or an another on the preferences of heterogeneous individuals that run or own the firm, this would make no difference in the firm's decision making, since due to the monotonicity of preferences more profit is better for everybody and consequently the profit maximizing production plan would be compatible with the shareholders' preferences and hence unanimously accepted. In that sense, the detachment of the firm's decision making from the preferences of its owners justifies the characterization of the firm as an autonomous entity.

When a firm operates in an uncertain environment, its activity involves a certain amount of risk. When the production technology is not instantaneous but evolves through time, an investment decision that realizes in the present sacrifices resources or inputs whose transformation to output depends on the possible events that may occur in the future. Consequently, future profit itself is variable across different state-events of the economy. The objective of the firm under uncertainty is to maximize its expected profit or equivalently to choose that production project whose present value is the highest among alternative candidates.

The evaluation of a project however, involves implicit prices or discount factors that translate future revenue in terms of present value. In perfect competition, implicit prices are provided by the stock or, more generally, the asset market. Whether such prices are unique or not is crucial in terms of firm's decision making. When a firm faces a multiplicity of discount factors compatible with competitive equilibrium, its choice becomes indeterminate.

The uniqueness of implicit prices of revenue depends on the nature of the asset market.

When the asset market is complete, given the equilibrium asset prices and returns, there exists a unique set of implicit state prices of revenue compatible with no-arbitrage asset prices, which is common across all individuals. In this economy, the competitive price mechanism renders the discounting of future revenue uniform across individuals and firms and the objective of the firm is well defined exactly as in the case of no uncertainty. The argument made earlier on the characterization of the firm as an autonomous entity, applies also here. Furthermore, under complete asset markets, any feasible production plan lies within the span generated by the existing assets and so it can be unambiguously priced by their equilibrium prices. The firm in this context cannot add any new risk sharing opportunity in the market since there are already enough assets available to fully insure against all contingencies.

When the asset market is incomplete, there exists a multiplicity of implicit prices of revenue compatible with equilibrium prices. The lack of uniqueness suggests that there does not exist enough objective market-based information to guide production decisions. Any two distinct vectors of state prices of revenue compatible with equilibrium prices that could be utilized by the firm to discount future profits would result in different production decisions. As a consequence, expected profit maximization is not well defined unless we can justify a decision rule in order to select among the multiple implicit prices compatible with equilibrium, the ones that should be used by the firm.

It is reasonable to assume that when the market does not provide sufficient information for the evaluation of production plans, the preferences of owners, which in an uncertain environment summarize their priors and attitude towards risk, would have to play an essential role in the firms' decision making. When a single individual owns a firm, his unique implicit prices of revenue can be used to define unambiguously profit maximization. On the other hand, when a firm is owned by a group of individuals new issues arise. In incomplete markets individuals maximize under a multiplicity of budget constraints imposing that expenditure across date-events be the payoff of a portfolio of marketed assets. Consequently, implicit prices of revenue across date-events are typically distinct among agents at equilibrium and thus individual evaluations of production plans will not coincide. Shareholders of a firm will have conflicting views over the production decision and the ownership structure of the firm will be relevant to that decision. The firm is no longer autonomous as in the certainty or the complete markets case but its choice is guided by the preferences of its shareholders and the ownership structure that determines the relative control power of its shareholders.

1.1 A Brief Overview of the Literature

In the literature[1] one can find various approaches to the firm's objective function that can be divided in two broad categories according to the source of information for the evaluation of production plans: information that relies on the market and information that relies on shareholders preferences.

In (Ekern S. and Wilson R. 1974), (Radner R. 1974), (Leland H. 1974), (Grossman S. and Stiglitz J. 1980), the production set of the firm is restricted to investment plans that belong to the span of the payoffs of marketed assets. Plans not in the span of marketed asset are excluded a priori and unanimity among shareholders is virtually guaranteed. (De Waegenaere A., Polemarchakis M. and Ventura L. 1995) suggested a projection technique to evaluate such production plans by approximating their payoffs with the payoff of portfolios of marketed assets. The set of equilibria there coincides with the one of (Duffie D. and Shafer W. 1987) where firms maximize the value of payoff of shares at some implicit prices of revenue.

In the above contributions, where information is extracted by the market, the ownership structure of the firm and the control rights implicitly attached to shares play no role to the production decision.

The problem of constructing an objective function for the firm based on shareholders' preferences is a collective decision problem. In (Dreze J. 1974) and (Grossman S. and Hart O. 1979) the firm discounts profits by aggregating individual marginal evaluations of production plans by its shareholders. Another viewpoint is to incorporate majority voting in firms' decision making. (Gevers L. 1974) explores the possibility of non-existence due to the Condorcet paradox. (Dreze J. 1985) proposes a model of control power within the firm and shows that existence of equilibrium is always insured by introducing a board of directors that has veto on the issues to be voted (agenda control). (Sadanand A. and Williamson J. 1991) demonstrate existence of voting equilibria when the majority rule is direction restricted, that is, shareholders vote for modifications of the production plan one dimension at a time. (DeMarzo P. 1993) shows that under certain assumptions when a voting equilibrium exists then the production plan is optimal for the individual that holds the largest amount of stock within a firm. (Kelsey D. and Milne F. 1996) extend the (Dreze J. 1985) model by introducing an infinite set of individuals and externalities. (Dierker E. et al. 1999) examine the role of non-convexities of the feasible set for the comparison of different partnership equilibria. In the presence of unlimited short-sales, (Momi T. 2001) shows non-existence of competitive equilibria in economies where firms produce according to a modified Dreze decision rule.

The question of Pareto optimality of the competitive equilibrium of a production economy with incomplete security markets was first raised by (Diamond P. 1967). He showed that if there is only one good and firms face multiplicative uncertainty, then every equilibrium allocation is constrained efficient[2]. (Dreze J. 1974) introduced a criterion for more general production functions and showed that constrained efficiency still obtains in the one good economy, however he also provided examples of inefficient equilibria caused by the non-convexity of the set of feasible allocations of production plans and asset portfolios. (Geanakoplos J. et al. 1990) showed that if there are two or more goods then the allocation of investment induced by the stock market is generically constrained inefficient. (Dierker E. et al. 2002) present examples of Dreze equilibria with fixed initial distribution of endowments and constant returns to scale technology that are all constrained inefficient.

2. A Control Based Investment Criterion

In the sequel we present an extension of the (Dreze J. 1974) model in order to capture the institutional aspects that relate to corporate control concerning production decisions. In particular, the investment criterion we propose requires that the production decision is endorsed by the majority of the initial shareholders, a requirement that originates in modern corporate law. It does not require unanimity. As a consequence, only the preferences of the initial shareholders with considerable control power are crucial in the construction of our corporate decision rule, whereas the preferences of minor shareholders are taken into account only to the extent that their votes are crucial in the decision process.

The model proposed by (Dreze J. 1974) is considered a benchmark in the literature of the theory of the firm under uncertainty. According to it, firms not only choose their production plans so that they are unanimously accepted by the shareholders (without requiring redistribution of the firm's output through transfers across shareholders), but the production choice being optimal at the firm level turns out to be optimal also at the economy level. The competitive allocation is constrained Pareto optimal.

A criticism about the Dreze model is that the informational requirements for the firm are too demanding. Indeed, the firm needs to know the discount factors of its shareholders in order to evaluate which plan is optimal[3]. Despite of the fact that there is no empirical evidence to what extent firms do collect information on the preferences of their shareholders concerning production decisions, it is reasonable to assume that preferences of at least a subgroup of firm's shareholders do matter in the decision-making.

The objective function of the firm suggested by (Dreze J. 1974) was designed as such on normative grounds in a perfectly competitive economy. The requirement that a production plan has to be unanimously accepted if it is to be implemented, is a very strong one. In particular, it neglects the fact that in corporations the absolute or relative majority decides, even against the preferences of the minority. There is no guaranty that a shareholder holding more than 50 per cent of a firm, will be willing to accept the production plan derived from the Dreze rule, if he can make use of his control right, which is institutionally established, and impose a different, but more beneficial for him, production plan. Although the market for shares is competitive, the control rights attached to shares give strategic power to the shareholders regarding the production decision of the firm, which cannot be taken explicitly into account by the market mechanism[4].

In order to capture the role of control power on production decisions, we present a variant of the Dreze model according to which firms use as implicit prices of revenue the weighted sum of the individual implicit prices, the weights being the Shapley values (of a simple majority game) of initial shareholdings of each firm owner. The difference with the Dreze objective is that only the preferences of initial shareholders with positive Shapley value are taken into account in the objective of the firm and not the preferences of each of the final shareholders. The objective proposed here is consistent with the fact that minor shareholders have no influence on the production decision of the firm, unless their vote is crucial into turning a losing coalition into a winning one in a single majority voting game.

2.1 The Economy

Economy extends over two time periods, $t = \{0,1\}$. There are S possible states of the nature indexed by $s \in S$, one of which will be realized at period 1. There are I individuals, $i \in I$, and J firms, $j \in J$, in the economy. Individuals apart from being consumers, they are owners of stock of the firms. We denote by β_j^i the share of firm $j \in J$ held by individual $i \in I$, $\beta_j^i \geq 0$, and $\beta^i = (\ldots,\beta_j^i,\ldots)$ a portfolio of shares[5]. We will assume that shares represent a percentage of ownership of each firm, so

$$\sum_{i \in I} \beta_j^i = 1, \forall j \in J. \tag{1}$$

There exists a unique commodity available in period 1, hence there are S state contingent commodities. Let $x^i = (\ldots,x_s^i,\ldots)$ by a consumption bundle of individual ι, where is consumption at state s. The consumption set of individual i is X^i R + ^{S}. At the beginning of period 0, an individual is endowed with a portfolio of shares β^i, the initial portfolio; at the beginning of period 1, he is endowed with a vector of state contingent commodities, $e^i = (...,e^i_s,\cdots)$ where e^i_s is the endowment of the unique commodity in state s. Preferences over consumption bundles are represented by strictly monotone, continuous and twice differentiable utility function, $u^i x^i)$.

A firm is characterized by its technology set Y^j c R^S. *A* production plan is denoted by y^j ε Y^j, $y^j = (...,y^j_s,...)$, where y^j_s is input/output in terms of the physical commodity in state *s*.

Assumption *(technology sets).* The technology set Y^j is closed and convex, $Y^j \supset \Re_-^S$ (possibility of free disposal and inaction), $Y^j \cap \Re_+^S = \{0\}$, there are no free goods in production, and the production possibilities of the overall economy are bounded, that is, $(\sum_i e^i + \sum_j Y^j) \cap \Re_+^S$ is compact for all $e^i \in \Re_+^S$.

A feasible production plan is a plan that belongs in the technology set of the firm. Firms choose a feasible contingent production plan in period 0. In period 1, after uncertainty is revealed production takes place.

In period 0, there exists a stock exchange where individuals trade their initial endowments of shares β^i at prices $q = (...,q^j,...)$, a row vector. The value of their *initial* portfolio of shares $q\beta^i$ is reallocated across shares of J firms. The after trade or *final* portfolio of shares of an individual is denoted by $\theta^i \in \Re_+^J$ and entitles a shareholder to $Y\theta^i$ units of the unique commodity produced in period 1. The matrix Y is an SXJ matrix, $S > J$, whose columns represent production plans of all firms,

$$Y = (y^1,\ldots,y^j,\ldots y^J) = \begin{bmatrix} y_1^1 \cdot y_1^j \cdot y_1^J \\ \cdots\cdots\cdots\cdots \\ y_s^1 \cdot y_s^j \cdot y_s^J \\ \cdots\cdots\cdots\cdots \\ y_S^1 \cdot y_S^j \cdot y_S^J \end{bmatrix}$$

The optimisation program of a consumer-shareholder is to maximize his utility function given the production plans of the firms Y and prices of shares q,

$$Max_{\theta}\ u(x^i) \tag{2}$$
$$\text{s.t. } q\theta^i \leq q\beta^i$$
$$x^i \leq e^i + Y\theta^i$$

The Lagrangean is

$$L^i = u^i(x^i) + \lambda_0^i \sum_{j\in J} q^j(\theta_j^i - \beta_j^i) + \sum_{s\in S} \lambda_s^i \left(x_s^i - e_s^i - \sum_{j\in J} \theta_j^i y_s^j\right),$$

and the solution to the optimization program is given by

$$q^j = \sum_{s\in S} (\lambda_s^i / \lambda_0^i) y_s^j\ , \tag{3}$$

which gives rise to a vector of marginal rates of substitution, $\pi^i \in \Re_{++}^S$, between income in period 0 and in period 1, state s,

$$\pi^i = (...,\pi_s^i,...) = (...,\lambda_s^i / \lambda_0^i\ ,...). \tag{4}$$

2.2 The Decision Rule of the Firm

Let by O^j denote the set of initial owners of firm j,

$$O^j = \{i \in I : \beta_j^i > 0\}.$$

We define a simple majority game as a set of players O^j and a characteristic function v defined on the subsets of O^j. Let ω^j be a non-empty subset of O^j; a coalition. Function v satisfies the following properties:

$$v(\varnothing)=0,$$

If $\omega^j \cap \overline{\omega^j} = \varnothing$ then $v(\omega^j \cup \overline{\omega^j}) \geq v(\omega^j) + v(\overline{\omega^j})$

and

$$v(\omega^j) = 1 \text{ if } \sum_{i\in\omega^j} \beta_j^i > ½,$$

$v(\omega^j) = 0$ if $\sum_{i \in \omega^j} \beta_j^i \leq 1/2$

The function v qualifies whether a coalition of shareholders ω^j is a winner, $v(\omega^j) = 1$, in an one share/one vote majority game where the shareholders vote to decide about the production plan of the firm.

The simplest way to represent this game is by the vector:

$[1/2;(\ldots,\beta_j^i,\ldots)]$, $i \in O^j$

where 1/2 denotes the number of votes per cent needed by a winning coalition and β_j^i, the number of votes per cent cast by shareholder $i \in O^j$.

The Shapley value $Sh\ [1/2;(\ldots,\beta_j^i,\ldots)]$ is an $|O^j|$ -vector $(\ldots,\varphi^i,\ldots)$ such that:

$$\varphi_j^i = \sum_{\omega^j : i \in w^j} \frac{(|\omega^j|-1)!(|O^j|-|\omega^j|)!}{|O^j|!} \tag{5}$$

where the summation is taken over all minimal winning coalitions ω^j i.e. winning coalitions none of whose subset $\omega^j - \{i\}$ is winning[6].

In fact, in our model there is no explicit voting. We use the Shapley value, (Shapley L. 1953), as a measure of the control power of a shareholder on the firm's decision or as an approximation of the outcome of a voting procedure. Assuming that each winning coalition has the same probability to be formed (symmetry axiom), we can measure the control power of every shareholder over the production outcome by summing his probabilities

$(|\omega^j| -1)!(|O^j| -|\omega^j|)!/|O^j|!$

to be pivotal, that is, to enter a formed loosing coalition, ω^j-$\{i\}$, such that $v(\omega^j\text{-}\{i\}) = 0$, and transform it into a winning one, $v(\omega^j) = 1$. In other words, the Shapley value measures the probability that a shareholder casts a decisive vote.

In period 0, the firm uses as implicit prices to evaluate future production, a weighted average of the individual marginal rates of substitution, and the weights are the Shapley values of each shareholder belonging in O^j. Given a set of implicit state prices of its initial shareholders, $(\pi^i)_{i \in O^j}$ and their respective Shapley values $(\varphi_j^i)_{i \in O^j}$, the problem of the firm is the following

$$\max \sum_{s \in S} \pi_s^j y_s^j \qquad (6)$$

st $y^j \in Y^j$

where

$$\pi_s^j = \sum_{i \in O_j} \varphi_j^i \pi_s^j \qquad (7)$$

So according to (7), the firm aggregates the preferences of its initial shareholders in order to evaluate production plans, but only the preferences of those individuals with a positive Shapley value will matter in its choice. Moreover, this decision rule favours major shareholders since their preferences, according to the Shapley value weight, will matter relatively more.

On the other hand, the control power of some shareholders, as measured by the Shapley value, may be less than the percentage of their shareholdings. Even though some shareholders hold a relatively large fraction of shares, their control power may be small if they are unable to form more winning coalitions than the other co-owners of the firm. In example II in section 3.2, a corporation; is owned by four individuals according to the following percentage distribution of shares across owners $(\ldots,\beta_j^i,\ldots) = (0.46, 0.05, 0.1, 0.39)$. Then according to the Shapley value rule (5) the distribution of control power will be[7]

$$(\ldots,\varphi_j^i,\ldots) = Sh\ [1/2;(0.46,\ 0.05,\ 0.1,\ 0.39)] = (1/2,\ 1/6,\ 1/6,\ 1/6)$$

It is clear that, compared to the number of shares or votes each shareholder initially possesses, more control power is attached to shareholders 1,2 and 3 since $\varphi_j^i > \beta_j^i$ for $i = \{1,2,3\}$ and less to shareholder 4 since $\varphi_j^4 < \beta_j^4$. Shareholder 1 is pivotal in six out of twelve winning coalitions whereas shareholders 2,3,4 have equal control power since they are pivotal in two winning coalitions. Despite of the fact that shareholder 4 owns the 39% of the firm his control power is only 16.66% since he can form the winning coalitions, {1,4} and {2,3,4}.

If a shareholder owns more than 50% of a firm, his weight φ_j^i equals to 1, and consequently the production plan that will be chosen by the firm will be optimal according to his preferences[8], since from (7), $\pi^j = \pi^i$, for i: $\beta_j^i > 1/2$. This case is illustrated in example II, in section 3.

2.3 Competitive Equilibrium

An allocation of endowments is $e^I = (...,e^i,...)$. An allocation (x^I,θ^I,y^J) is a vector of consumption plans $x^I = (...,x^i,...)$, a vector of portfolios $\theta^I = (...,\theta^i,...)$ and a vector of production plans $y^J = (...,y^j,...)$.

Definition 1 *An allocation (x^I,θ^I,y^J) is feasible if and only if, for every individual, $x^i \leq e^i + Y\theta^i$ and $\sum_{i\in I}\theta^i = \mathbf{1}_J$.*

Definition 2 *A competitive equilibrium $(q^*,(x^{I*},\theta^{I*},y^{J*}))$ is a pair of prices and a feasible allocation such that for every individual, $x^{i*} \in x^i(q^*,y^{J*})$, $\theta^{i*} \in \theta^i(q^*,y^{J*})$ and for every firm $y^{j*} \in y^j((\pi^{i*})_{i\in O^j})$.*

At equilibrium of a production economy we require that each shareholder chooses a portfolio of shares θ^{i*} that solves his individual maximization problem (2) given the optimal choices of the firms y^{J*} and the share prices q^*, and each firm chooses a feasible production plan y^{j*} that solves (6) given the implicit prices of its shareholders $(\pi^{i*})_{i\varepsilon O^J}$ evaluated at the optimal consumption point $x^{i*} = e^i + Y^*\theta^{i*}$rO <*.

2.3.1 Existence and Optimality of Competitive Equilibrium

Existence of competitive equilibrium for a production economy under uncertainty and incomplete markets has been proved in (Dreze J. 1974). The choice of particular weights for averaging individual discount factors $\pi^i \in \mathbf{R}_+^s$ does not interfere at all with the conditions on technology sets and utility functions that guarantee that equilibria are smooth functions of the parameters of the economy. Given assumption 1 and the assumptions on agents' utility functions, that is, strict monotonicity, continuity and differentiability, the existence proof in (Dreze J. 1974) applies equally in our model.

The work of (Geanakoplos J. and Polemarchakis H. 1986) has shown that competitive equilibria of an exchange economy with incomplete asset markets are generically constrained inefficient. Their result was extended by (Geanakoplos et al. 1990) for a production economy with many goods. In the one good economy, the (Dreze J. 1974) criterion is the only criterion according to which every equilibrium is constrained Pareto optimal.

The investment criterion proposed here results in constrained Pareto inefficient outcomes. Although a formal proof of inefficiency of competitive allocations is not provided, it is clear from the analysis in (Geanakoplos et al. 1990) that any criterion cannot improve efficiency unless the interests of the final shareholders, that is, shareholders who will receive the stream of firm profits, is taken explicitly into account, as it is the case in the (Dreze J. 1974) model. Inefficiency arises from two sources. First, there is misallocation of production across firms *(production inefficiency)* and second there is misallocation of shareholdings across individuals *(portfolio inefficiency).* A planner by reallocating portfolios across individuals and production plans across firms can achieve constrained Pareto optimality.

3. Examples

In this section, we present two examples of a production economy and we calculate equilibria according to the control-based criterion. Both examples correspond to the same economy, but differ with respect to the characteristics of agents, in particular the individual shareholdings. The examples capture two interesting cases of a firm's ownership structure. In example I, the firm is owned by an individual holding 51% of the shares and consequently all control power over the production decision belongs to a single agent. In example II, each individual holds less than 50% of the shares of the firm. In that case, even some minor shareholders are favoured in terms of control power.

The purpose of the examples is to show that the distribution of control power differs dramatically with respect to the initial distribution of shares or votes. As a consequence production decisions and equilibrium allocations will be different to the ones where firms decide according to the Dreze or the Grossman-Hart criterion. These differences are illustrated by direct comparison of the equilibrium allocations resulting from the three criteria.

The problem of the firm according to (Dreze J. 1974) is given by (6) where

$$\pi_s^j = \sum_{i \in O^j} \theta_j^i \, \pi_s^j$$

whereas according to (Grossman S. and Hart O. 1979)

$$\pi_s^j = \sum_{i \in O^j} \beta_j^i \, \pi_s^j \, .$$

3.1 Example I

The sets of individuals, firms and states of the economy are respectively $I = \{1,2,3,4\}$, $J = \{1,2\}$, and $S = \{1,2,3\}$. The technology set of firm 1 is

$\{y_1^1, y_2^1, y_3^1\} = \{1,0,0\}$ and that of firm 2 is $\{y_1^2, y_2^2, y_3^2\} = \{-k, 1, \sqrt{k}\}$ for $k \geq 0$. We denote a vector of share prices by $q = (1, q^2)$. We assume that individuals have no initial endowments of commodities $e_s^i = 0, \forall i \in I, s \in S$, but they are owners of firms 1 and 2 according to an initial portfolio vector $\beta^i = (\beta_1^i, \beta_2^i)$.

The utility functions of the agents and the initial shareholdings are given below:

$(\beta_1^1, \beta_2^1) = (1/4, 51/100)$, $u^1 = 3 \log x^1(1) + \log x^1(2) + \log x^1(3)$

$(\beta_1^2, \beta_2^2) = (1/4, 9/100)$, $u^2 = \log x^2(1) + \log x^2(2) + 3 \log x^2(3)$

$(\beta_1^3, \beta_2^3) = (1/4, 2/10)$, $u^3 = \log x^3(1) + \log x^3(2) + \log x^3(3)$

$(\beta_1^4, \beta_2^4) = (1/4, 2/10)$, $u^4 = 3 \log x^4(1) + 2 \log x^4(2) + 2 \log x^4(3)$

In Table 1 below, equilibria are calculated according to the Dreze, the control based and the Grossman-Hart criterion. Since individual 1 holds initially 51% of firm 2, the Shapley value of his shares is 1 and 0 for the remaining shareholders. So firm 2 decides by taking into account the preferences of individual 1 only. Indeed by comparing his utility level across the three equilibria, we conclude that individual 1, would never accept the production level according to the Dreze or Grossman-Hart rule that makes him worse off. Notice that individual 1 likes to consume more in state 1 than in state 3 and for that reason the level of investment is lower (in absolute terms) when the firm decides using exclusively his preferences.

TABLE 1

Drèze equilibrium	**Ind 1**	**Ind 2**	**Ind 3**	**Ind 4**
π^i	(0.25,0.55,0.48)	(0.25,0.28,0.72)	(0.25,0.55,0.48)	(0.25,0.55,0.61)
$(\vartheta_1^i, \vartheta_2^i)$	(0.46,0.23)	(0.14,0.23)	(0.22,0.24)	(0.18,0.29)
x^i	(1.55,0.23,0.27)	(0.26,0.23,0.26)	(0.54,0.24,0.28)	(0.32,0.29,0.34)
u^i	−1.45	−6.80	−3.30	−5.76
π^2, k, q	π^2 =(0.25,0.49,0.57)	k =1.32	q = 0.77	

(continues)

Control based equilibrium	**Ind 1**	**Ind 2**	**Ind 3**	**Ind 4**
π^i	(0.25,0.58,0.54)	(0.25,0.29,0.80)	(0.25,0.58,0.54)	(0.25,0.58,0.70)
$(\vartheta_1^i, \vartheta_2^i)$	(0.49,0.24)	(0.13,0.23)	(0.21,0.24)	(0.17,0.29)
x^i	(1.67,0.24,0.26)	(0.26,0.23,0.24)	(0.57,0.24,0,26)	(0.24,0.29,0.31)
u^i	−1.25	−7.06	−3.32	−5.85
π^2,k,q	π^2=(0.25,0.58,0.54)	k=1.16	q= 0.87	
Grossman-Hart equilibrium	**Ind 1**	**Ind 2**	**Ind 3**	**Ind 4**
π^i	(0.25,0.57,0.51)	(0.25,0.28,0.76)	(0.25,0.57,0.51)	(0.25,0.57,0.65)
$(\vartheta_1^i, \vartheta_2^i)$	(0.47,0.23)	(0.14,0.23)	(0.21,0.24)	(0.17,0.29)
x^i	(1.61,0.23,0.26)	(0.30,0.23,0.25)	(0.55,0.24,0.27)	(0.33,0.29,0.33)
u^i	−1.35	−6.92	−3.30	−5.80
π^2,k,q	π^2=(0.25,0.54,0.56)	k =1.25	q = 0.82	

All shareholders except individual 1 would prefer the Dreze criterion to the Grossman-Hart or to the control-based criterion. The table below presents an ordering of the three equilibria from the individual shareholder's point of view, where 1 denotes the most favourable equilibrium.

	Drèze	**Grossman-Hart**	**Control-based**
Ind. 1	3	2	1
Ind. 2	1	2	3
Ind. 3	1	2	3
Ind. 4	1	2	3

3.2 Example II

The economy in this example is identical to the one in the previous example, except from the individual shareholdings of firm 2 which are given by

$$(\beta_2^1, \beta_2^2, \beta_2^3, \beta_2^4) = (49/100,\ 5/100,\ 10/100,\ 39/100).$$

In Table 2, we calculate the equilibria that correspond to the ownership structure of firm 2 above.

TABLE 2

Drèze equilibrium	Ind 1	Ind 2	Ind 3	Ind 4
π^i	(0.25,0.54,0.43)	(0.25,0.27,0.65)	(0.25,0.54,0.43)	(0.25,0.54,0.80)
$(\vartheta_1^i,\vartheta_2^i)$	(0.43,0.21)	(0.14,0.21)	(0.18,0.20)	(0.25,0.38)
x^i	(1.37,0.21,0.26)	(0.23,0.21,0.26)	(0.43,0.20,0.24)	(0.42,0.38,0.48)
u^i	−1.94	−7.07	−3.89	−4.25
π^2,k,q	π^2=(0.25,0.49,0.62)	k =1.55	q = 0.70	
Control based equilibrium	**Ind 1**	**Ind 2**	**Ind 3**	**Ind 4**
π^i	(0.25,0.57,0.48)	(0.25,0.28,0.72)	(0.25,0.57,0.48)	(0.25,0.57,0.92)
$(\vartheta_1^i,\vartheta_2^i)$	(0.44,0.22)	(0.13,0.20)	(0.18,0.19)	(0.25,0.39)
x^i	(1.47,0.21,0.26)	(0.23,0.20,0.24)	(0.43,0.19,0,23)	(0.44,0.39,0.46)
u^i	−1.74	−7.32	−3.95	−4.22
π^2,k,q	π^2=(0.25,0.52,0.59)	k = 1.41	q = 0.79	
Grossman-Hart equilibrium	**Ind 1**	**Ind 2**	**Ind 3**	**Ind 4**
π^i	(0.25,0.55,0.45)	(0.25,0.28,0.67)	(0.25,0.55,0.45)	(0.25,0.55,0.84)
$(\vartheta_1^i,\vartheta_2^i)$	(0.43,0.21)	(0.14,0.21)	(0.18,0.19)	(0.25,0.39)
x^i	(1.40,0.21,0.25)	(0.23,0.20,0.25)	(0.43,0.19,0.24)	(0.43,0.38,0.47)
u^i	−1.88	−7.15	−3.91	−4.24
π^2,k,q	π^2=(0.25,0.54,0.61)	k =1.50	q = 0.73	

Individual 1 who is a major shareholder of firm 2, likes consumption in state 1 more than in state 3. He prefers a low quantity of input k in state 1 and a high selling price for his shares. The control based criterion assigns a weight of a 0.50, instead of 0.46 and 0.21 for Grossman-Hart and Dreze criterion respectively, on his preferences in the construction of the implicit state prices of the firm. Consequently the control based equilibrium results in a lower k and a higher share price q and turns out to be the most favourable equilibrium for shareholder 1 as compared to the other two equilibria.

As far as individual 4 is concerned, although his weight is 0.16 according to the control based criterion as opposed to 0.39 according to the Grossman-Hart and Dreze equilibrium, nevertheless his favourite equilibrium is the control based equilibrium since like individual 1, likes more consumption in state 1 than in state 3.

Individuals 2 and 3 would prefer the Dreze equilibrium to the Grossman-Hart, because they are both net buyers of firm 2 shares and the Dreze equilibrium results in a lower equilibrium share price than the other two equilibria. Shareholder 2 prefers a lower k, since he likes more consumption in state 3 than in state 1

The table below presents an ordering of the three equilibria from the individual shareholder's point of view, where 1 denotes the most favourable equilibrium.

	Drèze	Grossman-Hart	Control-based
Ind. 1	3	2	1
Ind. 2	1	2	3
Ind. 3	1	2	3
Ind. 4	3	2	1

4. Conclusion

As already stated in the introduction, expected profit maximization under uncertainty in incomplete markets is not a well defined objective for the corporate firm due to the absence of a unique discount factor for defining the net present value of a firm's profit. The approach of Dreze to this problem is to choose out of the set of implicit prices of revenue that are compatible with equilibrium, the ones that induce an unanimously accepted production plan that guarantees constrained Pareto optimality for the overall economy. The approach here serves the purpose of introducing the feature of corporate control in firm's decision making, which is institutionally established as a mechanism for resolving shareholders' conflict.

In the model presented, we proposed an investment criterion which takes into account the preferences of shareholders with considerable corporate control power, excluding the preferences of minor shareholders which according to corporate law have no influence on the production decision. The distinction between major and minor shareholders, in terms of the influence they may exercise on the production outcome, is obtained by the application of the Shapley value rule for n-person simple majority games. The control-based criterion, as compared to the Dreze criterion, does not require unanimity, which is a prerequisite for constrained Pareto optimality of the equilibrium allocation but contradicts practice in corporations' current decision-making. Also the control-based criterion diminishes dramatically the informational requirements for the firm concerning the preferences of its shareholders since only the preferences of initial shareholders with positive Shapley value are needed for the construction of the firm's discount factor.

By construction of the control-based investment criterion, the equilibrium production plan of the firm is optimal for the shareholder with the absolute majority of shares, when voting rights obey the one share-one vote rule. In the case where no shareholder holds more than 50% of a firm, it is suggested via the examples that the control-based equilibrium will be preferable to the Dreze or the Grossman-Hart equilibrium from the largest shareholder's point of view. Although from the economy's viewpoint, the Dreze decision rule is desirable, as it is constrained Pareto efficient, from the individual shareholder's viewpoint it may not be. The institutional arrangement giving power to major shareholders to make production decisions is a realistic feature embodied in the criterion proposed here.

Another feature of the control-based criterion is that it reduces to the standard profit maximization criterion when the asset markets are complete, since the state prices of the firm will be a convex combination of identical individual state prices of revenue. In this way, the irrelevance of corporate control issues under complete asset markets is justified.

Notes

1. For a survey of the literature on this topic see (Dreze J. 1982) and (Magill M. and Quinzii M. 1996).

2. An allocation is constrained efficient, when a planner who is constrained to use the existing assets in the market cannot improve upon it. The planner is permitted to reallocate wealth or goods across state of nature only within the set of feasible allocations generated by the existing asset structure.

3. Although, the recent work of (Chiappori P.-A. et al. 2000), suggests that the identification of individual preferences from equilibrium asset prices is possible under certain assumptions and hence the firm may be capable of extracting the individual implicit prices of revenue out of asset prices.

4. Modelling this situation in a general equilibrium framework would require opening a market for control rights, but then it turns out to be problematic to decouple the market for control from the market for shares.

5. All vectors are column vectors except otherwise stated.

6. The general Shapley formula for n-person games is

$$\varphi_j^i = \sum_{\omega^j} (v(\omega^j) - v(\omega^j - 1))$$

where summation is taken over all possible coalitions. In the class of simple majority games when summation is taken over minimal winning coalitions,

$v(\omega^j) = 1$ and $v(\omega^j\text{-}l) = 0$.

Notice also that $\sum_i \varphi_j^i = 1$.

7. Given an initial distribution of shares (0.46,0.05,0.1,0.39) across shareholders {1,2,3,4}, the winning coalitions in which each shareholder $i \varepsilon I$ is essential are:

for $i = 1$, {(1,2,3), (1,3,4), (1,2,4), (1,2), (1,3), (1,4)}

for $i = 2$, {(2,3,4), (1,2)}

for $i = 3$, {(2,3,4), (1,3)}

for $i = 4$, {(2,3,4), (1,4)}

The Shapley value φ^1 for each shareholder $i \varepsilon I$

$\varphi^1 = = 6/12 = 0.5$

$\varphi^2 = = 1/6 = 16.66$

$\varphi^3 = = 1/6 = 16.66$

$\varphi^4 = = 1/6 = 16.66$

So, Sh[1/2;(0.46,0.05,0.1,0.39)] = (1/2,1/6,1/6,1/6).

8. If there exists i: $\beta_j^i > 1/2$, then according to (5) $\varphi_j^i = 1$ and $\varphi_j^h = 0$ for h i, $h \varepsilon I$.

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