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SEPARABILITY OF FIXED INPUTS IN DIFFERENTIAL SYSTEMS

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Abstract

This paper develops a differential demand system which involves fixed production inputs without the assumption that these are separable (strongly or weakly) from the perfectly variable inputs. This allows the allocation decisions for the perfectly variable inputs to depend on the intensity of use of the fixed inputs. It also allows testing for separability of fixed inputs through simple parameter restrictions. The differential input demand system has been applied to the agricultural sector in Greece for the period 1961-97. The null hypothesis of separability between Land and Capital (fixed inputs) and all the variable inputs have been decisively rejected suggesting that the intensity of use of Land and Capital do affect the allocation decisions for the variable inputs (JEL Classification: C3, D21).

Key Words: Differential Systems, Fixed Inputs, Separability.

1. Introduction

In empirical demand studies researchers approximate unknown true technologies employing either the dual or the differential approach. The former involves an approximation in the space of variables (Diewert and Wales, 1987; Diewert, 1974) while the latter involves an approximation in the space of parameters (Theil, 1980; Laitinen and Theil, 1978, Rossi, 1984). Both approaches result into systems which posses the desirable properties of parsimony and flexibility. An advantage of the differential approach is that because it does not specify a particular form for the underlying technology, it may accommodate different technologies without being exactly appropriate for any particular one (Barten, 1993; Mountain, 1988).

The theory of production allows for fixity of certain inputs in the short-run. In dual models the use of fixed inputs is a very common practice in both the theoretical and the empirical research (e.g. Weaver, 1983;

Morrison, 1986; Dupond, 1991; Hauver, Yee, and Ball; Lansink and Stefanou, 1997). The issue of input fixity, however, has received much less attention in the literature of differential systems. Indeed, except for the work of Rossi (1984), all empirical applications of differential demand models have been carried out under the assumption that all production inputs are perfectly variable (e.g. Davis, 1997; Theil, 1977). In allowing for fixed inputs, Rossi specified a technology that is strongly (additively) separable in the fixed input vector. The implication of separability, in general, is that the marginal rates of technical substitution between the variable inputs (and the optimal variable input ratios) are independent of the level of the fixed inputs. In other words, under separability a firm's expansion path with respect to the fixed inputs is a straight line from the origin. The implication of strong separability, in particular, is that the marginal physical products of the perfectly variable inputs are invariant to the level of any fixed input (Chambers, 1989). Given that separability of fixed input imparts severe restrictions on a production technology a researcher has better test for its existence rather than impose it *a priori* on an empirical model.

The objective of this paper is to develop a differential demand system with fixed inputs without employing the separability assumption. In what follows, section 2 presents the theoretical framework, while section 3 develops the differential demand system with fixed inputs and the statistical tests for separability. Section 4 involves an empirical application of the model to the Greek agricultural sector for the period 1961-97, and section 5 offers conclusions.

2. Theoretical Framework

Let C be a variable cost function giving the minimum cost required to produce output level y, given the prices of variable inputs, p_i, i=1, 2, ..., n, and the stock of a fixed factor, k.

Formally,

$$1)\quad C(y,p,k)=\min_{q_i} \sum_i p_i q_i$$

subject to $y=f(q,k)$,

where f is a twice continuously differentiable, increasing in k and increasing and concave in q production function.

The elasticity of variable cost with respect to y is

$$2)\ \gamma = \frac{C_y y}{C},$$

where C_y is the marginal production cost. Differentiating both sides of (1) with respect to y yields,

$$3)\ C_y = \sum_i p_i \frac{\partial q_i}{\partial y}.$$

The share of the ith variable input in C_y, that is, the proportion of C_y allocated to this input (Theil, 1977), may be expressed as

$$4)\ \theta_i^y = \frac{p_i \frac{\partial q_i}{\partial y}}{\sum_i p_i \frac{\partial q_i}{\partial y}} = \frac{w_i e_{iy}}{\gamma},$$

where w_i and e_{iy} are the budget share and the output elasticity of the ith input, respectively. By definition,

$$5)\ \sum_i \theta_i^y = 1 \Rightarrow \gamma = \sum_i w_i e_{iy}.$$

$$6)\ s = \frac{C_k k}{C},$$

where C_k is the marginal shadow value of k (Hulten, 1986; Morrison, 1986). Differentiating both sides of (1) with respect to k obtains

$$7)\ C_k = \sum_i p_i \frac{\partial q_i}{\partial k}.$$

The share of the ith variable input in C_k is

where e_{ik} is the elasticity of the ith input with respect to k. By definition,

Consider now a proportionate increase in y and k. Then, from World's identity (Brown et. al, 1995) one obtains

10) $pC = C_y y + C_k k,$

where $p = \sum_{r=y,k} \frac{\partial 1nC}{\partial 1nr} = \gamma + s$ is the cost elasticity from this proportionate increase in k and y. The Right Hand Side of (10) in which y and k are evaluated at their respective marginal effects on C (C_y, and C_k) can be thought as the "Total Marginal Cost" from a proportionate change in the output and the fixed input.[2] We denote this "Total Marginal Cost" by $\varnothing$. The shares of y and k in $\varnothing$ are

11) $g^y = \frac{C_y y}{\varnothing} = \frac{\gamma}{\varrho}$ and

12) $g^k = \frac{C_k k}{\varnothing} = \frac{s}{\varrho},$

implying that these are simply the ratios of the individual cost elasticities to the total cost elasticity p. Finally, combining relations (11), (12), (4) and (8) one obtains the share of the ith variable input in $\varnothing$, denoted by θ_i as

13) $\theta_i = \frac{\sum_i p_i \frac{\partial q_i}{\partial r}}{\varnothing} = \sum_r \theta_i^r g^r$, r=k,y and i=1,2,... n.

Given relations (5) and (9) and the fact that $g^y+g^k=1$, it follows that $\sum_i \theta_i = 1$.

3. A Differential Model with Fixed Inputs

The demand function for the ith variable input is q_i—$q_i(y,k,p)$. Taking the total differential of it, multiplying through by p_i/C and utilising relations (4) and (8) obtains the allocation decision for the ith input as a function of changes in y, k, and in the input prices

$$14)\ w_i d\ln q_i = \gamma\theta_i^y\, d\ln y + s\theta_i^k\, d1nk + \sum_j \pi_{ij} 1np_j,\ i=1,\ 2,\ ...,\ n,$$

where $\pi_{ij} = w_i e_{ij}$ and e_{ij} is the elasticity of q_i with respect to p_j. Homogeneity of degree zero of q_i in p reruires $\sum_j \pi_{ij} = 0$, while symmetry implies $\pi_{ij}=\pi_{ji}$.

Considering a proportionate change in input prices and summing up the allocation decisions obtains

$$15)\ \sum_i w_i d\ln q_i = d\ln Q = \gamma d1ny + sd\ln k = p(g^y d\ln y + g^k d\ln k),$$

where dlnQ is the Divisia volume index of variable inputs. To relate dlnQ to the changes in variable cost C note that from $C = \sum_i p_i q_i$ follows $d\text{In}C = \sum_i w_i d\ln p_i + \sum_i w_i d\ln q_i$, suggesting that the Divisia volume index is nothing else than the percentage change in C net of the effect of changes in variable input prices. A well behaved variable cost function is strictly increasing in y (implying $\gamma > 0$) and strictly decreasing in k (implying $s < 0$) Therefore, relation (15) suggests that Divisia volume index increases (decreases) with y(k), ceteris paribus.

Adding and subtracting $\theta_i d1nQ$ in the Right Hand Sid of (14) and using (11), (12), and (15) yields, after some rearrangements.

$$16)\ w_i d\ln q_i = \theta_i d\ln Q + pg^y(\theta^i_y - \theta_i)d\ln y + pg^k(\theta^k_i - \theta_i)d\ln k + \sum_j \pi_{ij} d\ln p_j.$$

However, from relation (13) and the fact that $g^y+g^k=1$ follows

$g^y(\theta^y-\theta^i) + g^k(\theta^k_i-\theta^i)=0 \Rightarrow g^k(\theta^k_i-\theta_i) = g^y(\theta^y_i-\theta_i)$. As a result, relation (16) can be rewritten as:

$$17)\quad w_i d\ln q_i=\theta_i d\ln Q+pg^k(\theta^k_i-\theta i)d\ln(k/y)+\sum_j \pi_{ij}d\ln p_j,$$

or using relation (12), as:

$$18)\quad w_i d\ln q_i=\theta_i d\ln Q+s(\theta^k_i-\theta_i)d\ln(k/y)+\sum_j \pi_{ij}d\ln p_j,\quad i=1,2,...,n$$

which gives an alternative expression for the allocation decision as a function of the Divisia volume index, the change in the ratio of the fixed input to output, and the change in variable input prices. The adding up conditions for the system of equations (18) are

$$19)\quad \sum_i \theta_i=1,\quad \sum_i s(\theta^k_i-\theta^i)=0,\ \text{and}\ \sum_i \pi_{ij}=0.$$

When the output and the fixed input change at exactly the same rate the allocation decisions depend only on the changes in the total variable input volume and the changes in the variable input prices. The same, however, is true when y and k are separable from the viariable inputs since in this case the variable cost function has the form $C(p, k, y)= C(G(p), H(k,y))$ from which follows $\theta^r_i=\theta_i\ \forall\ i$, with r=k,y.[3] Thus, the term $s(\theta^k_i-\theta_i)$ captures the effect on variable input allocations of non proportional changes in k and y when these are not separable from the variable inputs. This effect is a pure substitution one since it is calculated holding the total variable input volume (and the variable input prices) constant. A positive $s(\theta^k_i-\theta^i)$ implies that a rise in the intensity of use of the fixed input (increase in k/y) leads to an increase in the allocations to the ith input or, equivalently, that the fixed input and the variable input i are net technical complements. The opposite is true when $s(\theta^k_i-\theta_i)$ is negative.

Given that the elasticity of variable cost with respect to the fixed input is negative, a positive $s(\theta^k_i-\theta_i)$ requires $\theta^k_i<\theta_i$, that is, the share of the ith input in the shadow value of k is smaller than the share of this input in the "Total Marginal Cost". The expression, $\xi_i = \frac{s(\theta_i^K - \theta_i)}{w_i}$ is an intensity elasticity giving the percentage change in the ith variable input incduced by an one percent change in the ratio k/y, holding the variable input prices prices and the total variable input volume constant. Extension to multiple fixed inputs is straightforward. The allocation decisions in this case are:

$$20)\ w_i d\ln q_i = \theta_i d\ln Q + \sum_{h=1}^{m} \varphi_{ih} d\ln k_h + \delta_{iy} d\ln y + \sum_{j=1}^{n} \pi_{ij} d\ln p_j,\quad i = 1,2,...,n$$

with $\varphi_{ih} = s_h(\theta^h{}_i - \theta^i)$ and $\delta_{ih} = \gamma(\theta^y{}_i - \theta^i)$.

In relation (20) s_h is the shadow value of the fixed input h, h=1,2,...m and $\theta^h{}_i$ is the share of the ith variable input in the "Total Marginal Cost" in the presence of m fixed inputs. Provided that all fixed inputs and the output do not change at exactly the same rate separability tests are possible. Specifically, when $\varnothing_{ih}=0$ for every i, the fixed input h is separable from variable inputs. Also, when $\varnothing_{ih}=0$ for every i and h all fixed inputs and the output are separable from the variable inputs. To see why the latter implies output separability recall that $\sum_{h=1}^{m} s_h(\theta^k{}_i - \theta_i) + \gamma(\theta^y{}_i - \theta_i) = 0$ which means that when the first term on the Right Hand Side is zero the second term on the Right Hand Side has to be zero as well, that is, changes in y do not affect the allocation decisions. The Wald statistic for testing separability of an individual fixed input follows the χ^2 distribution with n-1 degrees of freedom, while for testing separability of al fixed inputs and the output follows the χ^2 distribution with mx(n-1) degrees of freedom.[4]

4. An Empirical Application to Greek Agriculture (1961-97)

4a. The Empirical Model and the Data

For the empirical application the agricultural sector in Greece is modeled here as an aggregate competitive firm which produces one output using five perfectly variable inputs, namely, Labor (X1), Feed *(XT),* Fertilizers (X3), Chemicals (X4), and Fuel and Energy (X5) and two fixed inputs, namely, Land (Kl), and Capital (K2). The classification of inputs into perfectly variable and fixed which is adopted in the present study is similar to that employed in earlier empirical production studies in agriculture (e.g. Mergos, and Karagiannis, 1997; Coyle, 1992; Hauver and Yee, 1991; Weaver, 1983).

Prices indexes (1970=1), expenditure in current prices, and volumes (expenditure in constant 1970 prices) for Feed, Fertilizers, Chemicals, and Fuel and Energy has been obtained from the National Accounts of Greece (NAG). The volume of Labor has been obtained from the Eurostat publication "Economic Accounts for Agriculture and Forestry" and the publication of the European Commission "The Agricultural Situation in the Community". The wage index (1970=1) for Labor is available by the National Statistics of Greece (NSSG). This index along with information on agricultural wages from the Survey of Agricultural Holdings (carried out every four years from the NSSG) has been used to calculate expenditure on Labor. The stock of Capital (which includes both machinery and structures) is available by the NAG. Information on irrigated and non irrigated land is available by the NSSG. This, in conjunction with the respective rental prices provided in the study by Psarou (1994) and the Land price index (1970=1) which is available by the NSSG and the study of Chetui (1996) has been used to construct a "quality adjusted" Land variable as in the studies of Mergos and Karagiannis (1997) and Papanagiotou (1998). The output variable (Y) stands for the value of the agricultural production at constant (1970) prices and it has been obtained by the NAG. The data used in this study are available by the author upon request.

For the empirical application a Rotterdam parameterization has been adopted (Davis, 1997; Theil, 1980). This implies that all price, variable input volume, fixed input and output parameters appearing on the Right Hand Side of relation (20) have been treated as constants. The theoretical restrictions of symmetry and homogeneity have been imposed and the condition $\sum_{h=1}^{m} \varphi_{ih}+\delta_{iy}=0$ have been employed to introduce the intensity variables (k_h/y) as in relation (18). Constant terms have been appended to all equations to capture possible trend (technical change) effects on the allocation decisions (Clements, 1980).

Because of the adding-up conditions the error variance — covariance matrix of the five-equation model is singular, thus, one of the equations has to be omitted. Initial experimentation showed that the empirical results are invariant to the choice of the equation. Here, the Fuel and Energy equation has been dropped and its coefficients have been recovered from the theoretical restrictions (symmetry, homogeneity and adding — up). Finally, since discrete data are used for the empirical application the rates of change of the model variables have been approximated as $\ln(z_t)-\ln(z_{t-1})$, where ζ stands for a price or an intensity variable, while the cost shares have been approximated by 0.5* $(w_{it}+w_{it-1})$, as suggested by Moschini and Vissa (1993) and Lee et. al (1994). The estimation of the four-equation model has been carried out using the LSQ (Iterative SURE) procedure in the statistical program TSP 4.3.

4b. The Empirical Results

Table 1 presents the parameter estimates along with the corresponding asymptotic standard errors. The Hessian matrix of the substitution effects is negative semi definite as stipulated by the economic theory (the eigenvalues of the matrix are -0.133, -0.01, -0.009, -0.0013, and -0.00079). The DW are 1.83, 2.01, 2.34, and 1.66 for Labor, Feed, Fertilizers, and Chemicals equations, respectively, indicating absence of autocorrelation. The system coefficient of determination-calculated as $R^2_L = 1-1/(1+LR/T(n-1)$, where T is the number of observations, η is the number of estimated equations and LR is twice the difference between the log-likelihood of the estimated system and the log-likelihood of the same dependent variables regressed on constants only (Bewley, Young, and Coleman, 1987)-is 0.67. Given that the model is differential, the explanatory power of the variable input volume effects, the price effects, and the intensity effects together on the variation in the allocation decisions appears quite high.

The estimated system has been subjected to test for endogeneity. In differential input demand systems endogeneity may arise because of the presence of the change in total variable input volume, dlogQ, on the Right Hand Side of the input allocation decisions (Theil, 1976; Attfield, 1985). To test for this potential problem we resort to the theory of Rational Random Behavior (Theil, 1976 and 1980; Duffy, 1987) according to which dlogQ is exogenous when the disturbance covariance terms are proportional to the price parameters. Practically, for empirical differential models such as (20), exogeneity of the rate of change of the total variable input volume may be tested by confirming that $cov(u_i,u_j) = a\pi_{ij}$, where λ is a factor of proportionality and u's are the residuals of the estimated equations (Lee et. al, 1994). The regression of the residual covariances on a constant and on the price parameters gave $cov(u_i,u_j)$=0.12(2.7)-1342.2(76.3)5,y, with R =0.97, the numbers in parentheses being standard errors. The intercept of the above regression is insignificant while the slope is significantly different from zero suggesting that the residual covariances are indeed proportional to the price parameters. Hence, endogeneity does not appear to be a problem in the present study.

TABLE 1

Parameters Estimates and Asymptotic Standard Errors

Parameter+	Estimate	Standard Error	Parameter+	Estimate	Standard Error
b_1	-0.008	0.002*	b_3	-0.001	-0.0008
θ_1	0.42	0.059*	θ_3	0.02	0.021
π_{11}	-0.064	0.02*	π_{33}	-0.0085	0.0045**
π_{12}	0.061	0.017*	π_{34}	0.0001	0.002
π_{13}	0.014	0.006*	π_{35}	0.007	0.0035*
π_{14}	0.0018	0.003	φ_{31}	-0.043	0.018*
π_{15}	-0.013	0.006*	φ_{32}	0.06	0.017*
φ_{11}	0.051	0.052	b_4	0.001	0.0004*
φ_{12}	-0.11	0.05*	θ_4	0.018	0.01**
b_2	0.001	0.002	π_{44}	-0.03	0.01*
θ_2	0.46	0.058*	π_{45}	-0.001	0.002
π_{22}	-0.068	0.019*	φ_{41}	0.031	0.009*
π_{23}	-0.013	0.006*	φ_{42}	-0.016	0.01**
π_{24}	0.005	0.003	b_5	0.006	0.0009*
π_{25}	0.015	0.006*	θ_5	0.077	0.02*
φ_{21}	-0.046	0.05	π_{55}	-0.008	0.005**
φ_{22}	0.062	0.047	φ_{51}	0.008	0.019
			φ_{52}	0.031	0.025

*(**), Statistically significant at the 5(10) percent level or less, bi, i=, 1, 2, ..., 5 are the trend effects. 0ih (i = 1, 2, ..., 5 and h = 1, 2) are the coefficients associated with the intensity variables. The standard errors of the coefficients of the fifth equation (Fuell and Energy) have been calculated using the ANALYZ procedure in the TSP 4.3.

From the five total variable input volume parameters (θ_i) three are statistically significant at the 5 percent level or less, while one is significant at the 10 percent level or less. From the fifteen price parameters (π_{ij}) nine are statistically significant at the 5 percent level or less and two are statistically significant at the 10 percent level or less. From the ten intensity parameters (φ_{ih}) four are statistically significant at the 5 percent level or less and one is statistically significant at the 10 percent level or less. Finally, from the five trend (technical change) parameters (bi) three are statistically significant at the 5 percent level or less.

Table 2 presents the separability tests. The individual null hypotheses that Land and Capital are separable from the variable inputs are strongly rejected at any reasonable level of significance. The same happens with the joint hypothesis that the two fixed inputs and the output are separable from the variable inputs. On the basis of the empirical evidence one concludes that separability between the fixed and the variable inputs is not consistent with the technology of the Greek agricultural sector.

TABLE 2

Separability Tests

Null Hypothesis	Restrictions	Empirical Value*
Land is Separable for the Variable Inputs	$\varphi_{11} = \varphi_{21} = \varphi_{31} = \varphi_{41} = 0$	22.45 (0.0001)
Capital is Separable from the Variable Inputs	$\varphi_{12} = \varphi_{22} = \varphi_{32} = \varphi_{42} = 0$	23.095 (0.001)
Land, Capital and Output are Separable from the Variable Inputs	$\varphi_{11} = \varphi_{21} = \varphi_{31} = \varphi_{41} = \varphi_{12} = \varphi_{22} = \varphi_{32} = \varphi_{42} = 0$	35.46 (0.0002)

*, probability of observing higher value in parentheses. For the first two hypotheses the theoretical distribution is the Chi-Squared with 4 degrees of freedom while for the last hypothesis is the Chi-Squared with 8 degrees of freedom. Note that $\varphi_{51} = 0$, $\varphi_{52} = 0$ and $\varphi_{51} = \varphi_{52} = 0$ are true as long as the first, the second, and the third hypothesis, respectively, in Table 2 hold because of the adding — up conditions.

Table 3 presents the Divisia and the price elasticities for the perfectly variable inputs. The Divisia elasticities under the Rotterdam parametrization are $E_i = \frac{d\ln q_i}{d\ln Q} = \frac{\theta_i}{w_i}$ and give the precentage change in the variable input i induced by an one percent change in the total variable input volume, ceteris paribus. The highest Divisia elasticity is the one for Feed followed by those for Chemicals, Energy, and Labor. The Divisia elasticity for Fertilizers (although positive as expected) is not significant at the conventional levels. The price elasticities are $E_{ij} = \frac{d\ln q_i}{d\ln p_j} = \frac{\pi_{ij}}{w_i}$ and give the percentage change in the variable input i induced by an one percent change in the price of the jth invariable input, ceteris paribus. All own - and cross-price elasticities are substantially lower than unity implying inelastic demand responses to prices. The highest own-price elasticites are those of Chemicals, Feed, and Fertilizers while the lowest are those of Labor and Energy. The majority of cross-price elasticities are positive suggesting economic substitutability among the perfectly variable inputs in the sector. The only negative cross-price elasticities are those for Labor and Energy, Feed and Fertilizers and Chemicals and Energy.

TABLE 3

Divisia and Price Elasticities*

Divisia Elasticities		Price Elasticities				
Input/ Price		Labor	Feed	Fertilizers	Chemicals	Fuel/ Energy
Labor	0.72*	-0.11*	0.10*	0.025*	0.003	-0.02*
	(0.1)	(0.03)	(0.03)	(0.01)	(0.05)	(0.01)
Feed	1.83*	0.24*	-0.27*	-0.05*	0.02	0.06*
	(0.23)	(0.07)	(0.07)	(0.02)	(0.013)	(0.02)
Fertilizers	0.39	0.03*	-0.26*	-0.17**	0.002	0.015*
	(0.4)	(0.12)	(0.12)	(0.09)	(0.04)	(0.07)
Chemicals	1.22**	0.12	0.34	0.007	-0.39*	-0.07
	(0.64)	(0.21)	(0.22)	(0.13)	(0.12)	(0.13)
Fuel/Energy	0.76*	-0.13*	0.15*	0.07*	-0.01	-0.08**
	(0.21)	(0.06)	(0.07)	(0.03)	(0.02)	(0.048)

*(**), Statistically significant at 5(10) percent level or less. The elasticities calculated at the sample means.

Table 4 presents the intensity elasticities which are calculated as $\xi_{ih} = \frac{d\ln q_i}{d\ln(k_h / y)} = \frac{\varphi_{ih}}{w_i}$. As mentioned in Part 3, the intensity elasticities are pure substitution effects since they measure the percentage change in the demand for a variable input due to an one percent change in the intensity of use of a fixed input holding the total variable input volume and the variable input prices constant.

TABLE 4

Intensity Elasticities

Variable Input	Land	Capital
Labor	0.08	-0.19*
	(0.08)	(-0.08)
Feed	-0.18	0.25
	(0.2)	(0.18)
Fertilizers	-0.85*	1.2*
	(0.35)	(0.33)
Chemicals	2.07*	-1.04**
	(0.6)	(0.54)
Fuel/Energy	0.08	0.3
	(0.19)	(0.25)

*(**), Statistically significant at 5(10) percent level or less. The elasticities calculated at the sample means.

The Land input appears to be net technical complement for Labor, Chemicals and Fuel/Energy and net technical substitute to Fertilizers and Chemicals. The Capital input appears to be net technical complement for Feed, Fertilizers and Fuel/Energy and net technical substitute for Labor and Chemicals.

5. Conclusion

The differential input demand systems are flexible and parsimonious and provide approximations to an underlying unknown technology in the parameter space. Therefore, they constitute attractive alternatives to the dual models. Earlier theoretical and empirical works, however, either failed to recognize the existence of fixed production inputs or they employed the assumption of strong separability between the fixed and the perfectly variable inputs. This paper develops a differential demand system which involves fixed inputs without the assumption of separability (strong or weak). This allows the allocation decisions for the perfectly variable inputs to depend on the intensity of use of the fixed inputs. It also allows testing for separability through simple parameter restrictions.

The theoretical model has been applied to the agricultural sector in Greece for the period 1961-97. The null hypothesis of separability between Land and Capital (fixed inputs) and all the variable inputs has been decisively rejected at any reasonable level of significance suggesting that the intensity of use of Land and Capital do affect the allocation decisions (optimal variable input ratios). On the basis of the relevant intensity elasticities, the increase in the use of Land appears to work towards the expansion of Chemicals and the reduction of Fertilizers while the increase in the use of Capital increases the allocations to Fertilizers and reduces the allocations to Labor and Chemicals.

Since the mid-1980s land restrictions has been among the policy instruments applied throughout the E.U. in an effort to curtail rises in the agricultural supply. The empirical results of this paper suggest that for the case of Greece land restrictions had mixed effects vis-a-vis the environment since on the one hand they favored the use of Fertilizers (a technical substitute for Land) but on the other hand they discouraged the expansion of Chemicals (a technical complement to Land). The various types of investment aid to farmers, had mixed effects vis-a-vis the environment as well since they favored Chemicals but not Fertilizers. The increase in the intensity of Capital use, however, had a clear negative impact on the allocations to Labor (a technical substitute) contributing, thus, to the exodus of the agricultural population which characterizes the agricultural sector of Greece since the 1960s.

Notes

1. For the sake of simplicity we consider here a single fixed input. Multiple fixed inputs are considered in the next section.

2. For a similar characterization of the "Total Marginal Cost", in the context of a differential system with multiple outputs but without fixed inputs see Theil (1980, pp. 44).

3. When $C(p,k,y)=C(G(p),H(k,y))$, $\theta^k_i=\theta^y_i=\dfrac{pi\dfrac{\partial q_i}{\partial H}}{C_H}$. Using the last and relation (13) results into $\theta^k_i=\theta^y_i=\theta_i$, for ever i.

4. It is (n-1) and mx (n-1), respectively because from the adding up restrictions $\sum_{i=1}^{n}\varphi_{ih}=0$ which implies that if the n-1 parameters are zero the nth has to be zero as well.

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