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MONEY, «LAISSEZ-FAIRE» AND THE UNDERGROUND ECONOMY*

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Abstract

The fact that the use of money enables underground transactions to be kept undetected by the authorities, implies that its variation changes this ability too, thus imposing transaction costs. Variations in money alter in addition the interest rate and hence, the opportunity cost of holding money, which is another money related factor affecting underground economy. The present paper argues that this double role of money makes it useful in controlling underground activities, more useful than a campaign against tax evasion, which constitutes a motive for going underground. The whole discussion evolves around this thesis, tackling it analytically from different points of view (JEL Classification: E49, E59, H 26).

1. Introduction

Underground economy (economic activity unrecorded by the national accounts) and tax evasion (income not reported to the tax authorities) are phenomena experienced by all economies. Apart from scientific reasons, our interest in them comes also from the fact that in our imperfectly competitive world, the government's resources would be larger were these two phenomena to be absent (see e.g. Chang, Lai, Chang, 1999). The authorities tend to be in a constant pursuit of those who do not pay taxes because of either tax evasion or income generated underground, though the two acts may go hand in hand . That is, the authorities proceed on a case by case basis, running after both tax evasion and underground activities. At the individual level, such an "all-in policy" may indeed be correct. At a more general level, however, i.e. at the macroeconomic level, this paper argues that, in so far as the goal of minimizing underground activities is concerned, fighting

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tax evasion may end up undermining this goal even if the entrance into the underground economy is dictated by tax evasion purposes. This thesis may sound strange, but as Bhattacharyya of the University of Leicester pointed out in a 12 August 1997 issue of Wall Street Journal, it is backed by ample empirical evidence. It is in fact the empirical observation that has given rise to such a thesis. This paper derives it theoretically, thus corroborating the view that monetary policy and a laissez-faire mentality with regard to underground economy (e.g. tax reductions, less business regulations), are more effective in controlling this economy than a campaign against tax evasion.

The reason is that the volume of transactions in this economy depends on the availability of money as this availability affects the interest rate and underground transaction costs. It is assumed that a higher interest rate raises the opportunity cost of holding money and hence, the opportunity cost of entering or continuing operate in the underground economy. Entrance or continuation of operation implies underground investment, I_U, and is thereby modeled by postulating that $\frac{dI_U}{di} < 0$, where i is the interest rate[2]. Thus, an increase in underground transactions stemming from an increase in the tax rate, will raise the demand for money and hence, the interest rate, ceteris paribus, thus damaging the underground economy. Next, note that underground transactions are based mainly on cash in order to avoid detection from the authorities , and that when a certain volume of money is put in circulation, it is common knowledge that most of it will be held underground . This implies ceteris paribus that $\Delta M = k\Delta Y + n\Delta Y_U$, with $n = k\zeta$, $\zeta > 1$, where M,Y, and Y_U are money, official output, and underground output, respectively, while Δ denotes change; (later we shall introduce into the money demand equation, the interest rate, too). ζ reflects how well money serves underground transactions, or put differently, the additional money needed underground to carry out the same amount or transactions as in the official economy and keep them secret: Increased transaction (secrecy) costs impose increases on ζ to take care of them. In this manner, given money supply, a tax-rate-change induced increase in underground transactions, will raise the underground demand for money, which in turn will raise not only the interest rate but transaction costs as well, against underground economy.

We think that these are issues that deserve some attention especially when the role of the. monetary sector for matters underground has not been explored adequately by the literature, at least within the context advanced here. In what follows, the next section investigates diagrammatically the connection between the financial sector and underground economy, through the use of the Moore-Palley model of this sector. Central to this section is that, in view of the evidence about "the case of missing money", the government is aware that a considerable part of an increase in money supply will go underground . Based on the results of the same section, the subsequent one develops a macromodel of the overall, official and underground, economy that recognizes also the impact of tax evasion. Transaction costs are introduced then in section 4 in order to clarify the conclusions of section 3, to account for the cyclical behavior of the economy, and to confirm the dominant role of money vis a vis taxation as a means of influencing affairs in the underground economy. Section 5 concludes this paper with some additional remarks on the subject.

2. The Interest Rate and the Underground Economy

Consider the four panels of Fig. 1 where the discount rate, i_B, is assumed to be fixed, and the monetary authority manipulates only the monetary base, H . It follows that the lending interest rate, i_L, is also fixed and, of course, higher than the discount rate, the difference being determined by the profit maximizing behavior of banks. Thus, in panel (a), the loan supply curve, L_s, is perfectly elastic at the fixed i_L, and the equilibrium amount of loans is determined by the loan demand curve, L_d. In panel (b), the slope of the ray from the origin of the axes, gives the loan-demand deposits ratio, L/D. This ray is positively sloped reflecting the operation of the deposit-expansion multiplier. The ray $(H/D)T$ of panel (c) gives the monetary base-demand deposits ratio in the official economy and has positive slope too, given that the demand for H and D move in the same direction. This is the ray contemplated by the government whenever it does not take into account the fact that part of the economy's cash is used underground. Finally, panel (d) depicts the supply curve of the monetary dace, H_s, which is perfectly elastic at the fixed discount rate.

This at least is the situation in the financial sector of the economy if one ignores the presence of the underground economy. But, underground activities do exist and are based on cash exchange, which means that at $L=L^O$ and $D=D^O$, only H^O_F of H_T^O is used in the official economy. In addition, if $L^O{}_d$ shifts to Ld^1 ,only $H^1{}_F$ will be related with the official economy; the rest of H^1_T will be used for underground transactions. That is, given the volume of underground transactions, the expansion of the monetary base expands the amount of cash available for such transactions, thus making them easier and perhaps encouraging (increasing) them . This conclusion is not something new; it is readily obtained through the observation that underground transactions are based mainly on cash. Nevertheless, it has been obtained here within the context of fixed interest rates. This prompts the question whether changing these rates can mitigate the expansion of money supply and hence, the increased availability of cash for underground purposes, satisfying simultaneously the higher demand for loans.

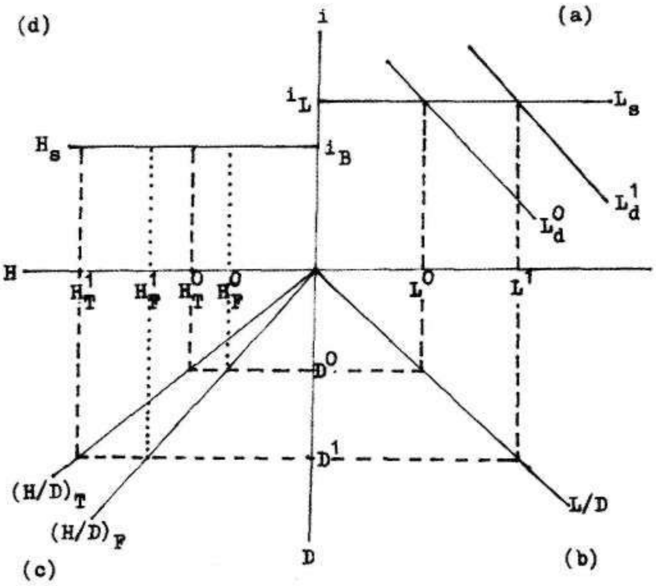

FIGURE 1.

As Fig. 2 shows, the answer is positive, ceteris paribus. This diagram is identical with Fig. 1 except in that the loan and base supply curves are now positively sloped while the rays in panels (b) and (c) have been rotated in the direction indicated by the arrows. The change in the slopes of L_s and H_s is one reason why the expansion of cash balances in the underground economy is moderated, since the increase of the (discount rate and hence, of the) interest rate serves as a means of only partially accommodating the increased demand for the monetary base. The other reason is the rotation of the above rays, rotation that results from the portfolio reallocation effect of the increasing interest rate.

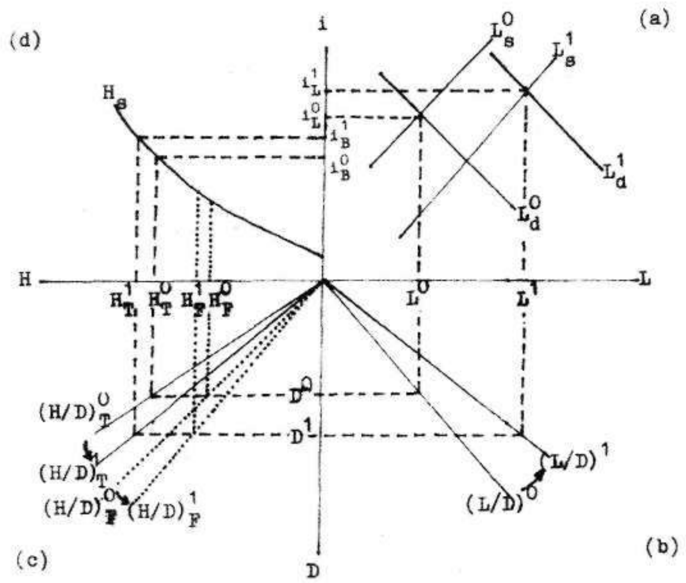

FIGURE 2.

This increase raises simply, the opportunity cost of holding cash and brings about eventually a rightward shift of the loan supply curve. Note that this cost is raised not only in the official economy but also in the underground economy to the extent that the return from certain underground activities becomes more or less similar to the return from e.g. time deposits. This is important, because raising the opportunity cost of underground activities will reduce some of them. Moreover, partial accommodation means partial expansion of cash underground and hence, an increase in underground transaction's cost relative to the case of a fixed interest rate. This is of great significance too, because the transactions costs literature tells that small transaction costs could induce serious output falls (see e.g. Leslie, 1993). In sum, from the viewpoint of policy, interest rate variations and transaction cost considerations can make underground activities more difficult.

3. Money, Interest, and Tax Evasion

According to our discussion thus far, an increase of the interest rate hurts the underground economy. Also, according to Figs 1 and 2, were the interest rate to decline, the reduction of cash balances would be smaller than that under a fixed interest rate. It appears that there is an inverse relationship between interest and underground economy.

Consequently, letting ΔY_U designate the change in underground output, we may safely assume that

$$\Delta Y_U = h - g\Delta i \quad (1)$$

where h and g are positive parameters. It might be argued for example, that if θ, $0<\theta<1$, out of a given Y_U is consumed and if the relationship $\Delta I_U = \lambda + \mu\Delta Y_U - \gamma\Delta i$ describes the change in underground investment (with $\mu,\gamma>0$ and with λ capturing the possibility of autonomous I_U) , then

$$\Delta Y_U = \theta\Delta Y_U + \mu\Delta Y_U - \gamma\Delta i + \lambda \; => \; \Delta Y_U = \frac{\lambda}{(1-\theta-\mu)} - \frac{\gamma}{(1-\theta-\mu)}\Delta i \quad (2)$$

and hence, $h=\lambda/(1-\theta-\mu)$ and $g=\gamma/(1-\theta-\mu)$. Note that $\Delta I_U/\Delta i < 0$ reflects the fact that an increase in the official lending rate, raises the underground rate, too. Also, in the absence of autonomous I_U, i.e. when $\lambda=0$, $h=0$ too, and (1) becomes just $\Delta Y_U = -g\Delta i$ (1), which is an expression that can simplify our discussion without loss of analytical insight. Of course, underground output is influenced by other factors as well, and notably, by tax evasion. Petersen (1990), for instance, postulates, ceteris paribus, that $\Delta Y_U = \delta\Delta t$, where t is the tax rate and δ is some positive constant. The underground activity increases as a means of tax evasion when the tax rate goes up . Therefore, one should write more accurately that:

$$\Delta Y_U = s\Delta t - g\Delta i \quad (3)$$

where $s>0$.

Absence of autonomous investment is assumed with respect to the official economy too, again for reasons of simplicity. In this economy,

$$Y=(e+j)(1-t)Y-vi+G$$

$$\text{or } \Delta Y=(e+j)(1-t)\Delta Y-(e+j)Y\Delta t-v\Delta i+\Delta G \quad (4)$$

where Y is official output, G is government expenditure, e denotes the marginal propensity to consume out of Y, and j and v are the positive constants of the "official" investment function I=j(l-t)Y-vi. Solving (3) for At and inserting the result in (4) yields

$$\Delta Y=(e+j)\ (1\text{-}t)\Delta Y\ -\ \frac{(e+j)Y}{s}\Delta Y_u-\left[\frac{(e+j)gY}{s}+v\right]\Delta i+\Delta G \tag{5}$$

From the monetary sector of the (overall) economy, we know that

$$\Delta M=k\Delta Y+n\Delta Y_U\text{-}m\Delta i\ \Rightarrow\ \Delta i=\frac{k}{m}\Delta Y+\frac{n}{m}\Delta Y_U-\frac{1}{m}\Delta M \tag{6}$$

where M is money while k, η and m are positive parameters . Hence, substituting (6) in (5), one obtains after some rearrangements that

$$\begin{aligned}\Delta Y_u=&\frac{(e+j)gY+vs}{(e+j)(1+ng)Y+nvs}\Delta M+\frac{sm}{(e+j)(1+ng)Y+nvs}\\ &\Delta G-\frac{sm-sm(e+j)(1-t)+vsk+(e+j)kgY}{(e+j)(1+ng)Y+nvs}\Delta Y\end{aligned} \tag{7}$$

As it was expected, increases in M and/or G increase Y_U, since a larger money stock makes underground transactions easier while higher public expenditures presuppose either more taxation or increases in money supply. This influence is offset by the fact that, ceteris paribus, $\Delta Y_U/\Delta Y<0$; which is also implied by the "stylized fact" that the fluctuations of Y_U and Y are of opposite direction (as it will be shown here too, in a while). For example, if e=0.75, v=4, j=Q.1, k=0.25, n=0.5, m=10, t=0.2, g=0.2, s=0.2, ΔM=30, ΔG=20, and Y=1000, then a ΔY=100, i.e. a 10% increase in Y, will lower eventually Y_U by 0.89 units[11].

What would be of more interest, however, is the responsiveness of Y_U to changes in s and g. To see this, assume that changes are infinitesimal, and integrate both sides of (7) to obtain the relation[12]

$$Y_u\frac{(e+j)gY+vs}{(e+j)(1+ng)Y+nvs}M+\frac{sm}{(e+j)(1+ng)Y+nvs}G-\frac{sm(1+ng)[1-(e+j)(1-t)+vsk}{(e+j)(1+ng)^2}$$

$$\log[(e+j)(1+ng)Y+nvs]-\frac{kgY}{(1+ng)} \tag{8}$$

Differentiating next (8) with respect to s and g yields

$$\frac{dY_U}{ds}=\frac{(e+j)Y}{[(e+j)(1+ng)Y+nvs]^2}[vM+(1+ng)mG]-$$

$$-\frac{m(1+ng)[1-(e+j)(1-t)+vk}{(e+j)(1+ng)^2}\{\frac{nvs}{(e+j)(1+ng)Y+nvs}+$$

$$+\log[(e+j)(1+ng)Y+nvs]\} \tag{9}$$

and

$$\frac{dY_U}{dg}=\frac{(e+j)Y}{[(e+j)(1+ng)Y+nvs]^2}[(e+g)YM-smnG]+\frac{smn(1+ng)[1-(e+j)(1-t)+nvsk}{(1+ng)^2}$$

$$\left\{\frac{\log(e+j)(1+ng)Y+nvs}{(e+j)(1+ng)}-\frac{Y}{(e+j)(1+ng)Y+nvs}\right\}+\frac{k}{(1+ng)^2}$$

$$\left\{\frac{nvs\log(e+j)(1+ng)Y+nvs}{(e+j)(1+ng)}-Y\right\} \tag{10}$$

To evaluate the signs of (9) and (10), note that (7) and our arithmetic example give ΔY_U 0 iff ΔM 25.4 when $G=0$, and ΔG 2167 when $\Delta M=0$[13]. This simply reflects the greater responsiveness of Yu to changes in M rather than G. Increasing g from 0.2 to 1.0, this responsiveness becomes even larger, because ΔY_U 0 iff ΔM 25.1 when $\Delta G=0$, and ΔG 10677 when $\Delta M=0$. We also conclude from these numbers that $\Delta Y_U/\Delta g>0$, since at the same G(M) it takes less M(G) to bring about the same ΔY_U as under $g=0.2$. Consequently, at the same M and G as under $g=0.2$, Y_U increases when g becomes equal to 1.0 and hence, $\Delta Y_U/\Delta g>0$.

A similar situation emerges when s changes from 0.2 to 1.0, but only for combinations of M and G exceeding 24.9 and 34, respectively; otherwise

$\Delta Y_U/\Delta s<0$. Noting, however, that $\Delta Y_U>0$ iff $\Delta M>26.8$ when $\Delta G=0$, and iff ΔG 467 when $\Delta M=0$, the "normal" case appears to be $\Delta Y_U/\Delta s>0$; in the Y_U-s space, we would expect a J rather than U pattern. An interpretation of this behavior of s is that as the responsiveness of Y_U to changes in taxation increases, the demand for money increases too, in order to finance the additional underground transactions, thus raising the interest rate against initially, but only partly cancelling out later, the expansion of underground activities . At the other end, as the responsiveness of Y_U to changes in the interest rate increases, more income is revealed to the authorities and is taxed at higher rates, which rates in turn increase eventually underground activity. Taxation turns out to be quite powerful in expanding the underground economy, perhaps because the changes of G needed to alter Y_U are very large vis a vis the change of M associated with the same ΔY_U. Therefore, from a policy viewpoint, rather that trying to increase the amount of income reported to the tax authorities (by influencing g upwards and/or fighting tax evasion), the policymaker should allow the interest rate to fluctuate freely so that it can increase and choke down the expansion of Y_U related with public sector matters . It is a policy of "don't do nothing in so far as the underground economy is concerned". It is from this perspective that the conclusion of the previous section should be appreciated.

4. Further Considerations

To understand the nature of our results further, recall that ζ and hence, $n=(k\zeta)$, $\zeta>1$ reflect the presence of underground transaction costs. Dividing now both sides of (7) by $\Delta\zeta$, the resulting expression becomes negative, because (a) $\Delta M/\Delta\zeta<0$, (b) as long as $\Delta G=\Delta M+\Delta T$, $\Delta G/\Delta\zeta<0$, T being total tax receipts, (c) a decline in ζ implies easier underground transactions, thus hurting the official economy, i.e. $\Delta Y/\Delta\zeta>0$, given that the coefficients of these derivatives are all positive. That is, $\Delta Y_U/\Delta\zeta<0$, which in conjunction with the fact that:

$$[(e+j)gY+vs]/[(e+j)(l+ng)Y+nvs]=d\log[(e+j)(l+ng)Y+nvs]/dn=(l/k)$$

dlog[(e+j)(l+ng)Y+nvs]A^, and that (9) and (10) can be rewritten so as to involve either of these two derivatives, the discussion following (9) and (10) should also take into account the influence of transaction costs. For example, increased taxation induces more underground transactions, which will be halted not only because the increased demand for underground money raises the interest rate (ceteris paribus), but also because the absence of more underground money increases transaction costs.

From still another perspective, note that according to (7) or (8), Y_U decreases at an increasing rate as income in the official economy expands, ceteris paribus. Indeed, when $\Delta M=0=\Delta G$,

$$\frac{d^2Y_U}{dY^2}=\frac{(e+j)kgnvs}{[(e+j)(1+ng)Y+nvs]^2}>0$$

That is, in the absence of money and public spending (and hence, taxes), underground economy would be a phenomenon of underdevelopment and would rapidly disappear with growth. It is the presence of M and G that keeps it "alive and well", since M reduces transaction costs considerably while G induces tax evasion by engaging in the underground economy. The sign of the derivatives

$$\frac{d^2Y_U}{dMdY}=-\frac{vs(e+j)}{[(e+j)(1+ng)Y+nvs]^2}<0$$

and

$$\frac{d^2Y_U}{dGdY}=-\frac{sm(e+j)(1+ng)}{[(e+j)(1+ng)Y+nvs]^2}<0$$

explains that this power of M and G declines as the official economy grows, but the relation $|(dYu/dM)+dYu/dG| > | dY_U/dY |$ remains a fact given that it implies from (7)

$(e+j)gY+vs+sm > sm-sm(e+j)(1-t)+vsk+(e+j)kgY$

or $[(e+j)gY+vs](1-k) > -sm(e+j)(1-t)$

which is always true, because $k<1$ and $-sm(e+j)(1-t)<0$. Moreover, although both M and G matter, the signs of (9) and (10) suggest that M matters more than G. Its connection with transaction costs makes it nonneutral, and to this type of nonneutrality, New Keynesians have ascribed even the emergence of business cycless (see e.g. Dore, 1993).

This is not to say that underground output cycles are generated only by transaction costs and add to the cycles of the official output. On the contrary, we shall show that the former cycles tend to alleviate the latter type, thus establishing further the negative relationship between Y_U and Y; and in order to do that, we presume destabilization by transaction costs, since only this factor can give rise to fluctuations in Y_U in our modeling, disturbing subsequently the intertemporal course of Y as well . A simple transformation of (8) would suffice to show the stabilizing influence of the underground economy (even in the absence of stabilizing manipulation in M and G). Setting:

$\Gamma = \{sm(1+ng)[1-(e+j)(1-t)]+vsk\}/[(e+j)(1+ng)^2]$, $K=(e+g)(1+ng)$, $\Lambda = nvs$ and $\Pi = (kg)/(1+ng)$, Y_U may be rewritten from (8) as follows:

$$Y_U = -\Gamma \log(\Lambda+ZY)-\Pi Y$$

or $Y_U = -\Gamma \log[\Lambda+Z\exp(\log Y)]-\Pi\exp(\log Y)$

or $Y_U = -\Gamma \log\{\Lambda+Z[\sinh(\log Y)+\cosh(\log Y)]\}-\Pi[\sinh(\log Y) + \cosh(\log Y)]$ (11)

where "exp" is the base of naperian logarithms while "sinh" and "cosh" designate the hyperbolic sine and cosine, respectively. We can find numbers Ψ and ω such that $\Psi\sinh(\omega)=1=\Psi\cosh(\omega)$ and hence, (11) becomes

$$Y_U = -\Gamma \log\{\Lambda+Z\Psi[\sinh(\log Y)\sinh(\omega)+\cosh(\log Y)\cosh(\omega)]\}-$$

$$-\Pi\Psi[\sinh(\log Y)\sinh(\omega)+\cosh(\log Y)\cosh(\omega)]$$

or $Y_U = -\Gamma \log[\Lambda+Z\Psi\cosh(\log Y+\omega)]-\Pi\Psi\cosh(\log Y+\omega)$

or $Y_U = -\Gamma \log\{\Lambda+Z\Psi\cos[\iota(\log Y+\omega)]\}-\Pi\Psi\cos[i(\log Y+\omega)]$

where «cos» is the circular (trigonometric) cosine and $\iota = \sqrt{-1}$. That is, the cycles of Y_U counteract the cycles of Y. Logically, as soon as $dY_U/dY<0$, Y_U should decline during the expansion of Y and increase during the contractions; it stabilizes the overall economy.

A government, of course, would prefer to have a $Y_U=0$ and to try stabilize the economy by itself, if stabilization is in the first lace desirable. Let us here confine policymaking to the manipulation of M and G that is necessary to achieve $Y_U=0$ under conditions of uncertainty. If E denotes the expectations operator, the problem in hand is to minimize E[(Yu-Yu)] with Y_U^* =0 and subject to the constraint provided by (8). Although it is reasonable to anticipate the presence of uncertainty, it is difficult to obtain the mathematical expectation of (8) because of the complex way Y is involved in that expression. Therefore, we shall assume that the authorities contemplate policymaking towards next period's Y_U after having observed this period's Y and subsequently, by treating Y as a constant and making (8) look as follows:

$$Y_U=AM+BG-W+\varepsilon \tag{8'}$$

where $A=[(e+j)gY+vs]/(\Lambda+ZY)$, $B=(sm)/(\Lambda+ZY)$ and $W=\Gamma \log (\Lambda + ZY) + \Pi Y$, while ε is a random disturbance term with $E(\varepsilon)= \varepsilon$ and variance given by σ_ε^2 . This random term is entered into (8') additively, but we shall assume the presence of multiplicative uncertainty too, via random disturbances in the coefficients A and B as well as in W. Such disturbances render the impact of the policy variables stochastic, as is the case in reality.

Inserting (8′) in $E(Y^2_U)$ and letting $E(A)=\overline{A}$, $E(B)=\overline{B}$ and $E(W)=\overline{W}$, the relationship that has to be minimized becomes

$$E[(AM+BG-W+\varepsilon)^2]=\sigma^2_A M^2+\sigma^2_B G^2+2\sigma_A\sigma_B pMG-\sigma^2_W-\sigma_\varepsilon^2 + (\overline{A}M+\overline{B}G-\overline{W}+\overline{\varepsilon})^2 \tag{12}$$

where p is the correlation coefficient between A and B; these coefficients share the same denominator. We should have also postulated correlation with W, but it would complicate our results unnecessarily for what we want to say concerning the relative intensity of the two policy instruments, M and G. Thus, from the first-order conditions, we derive the ratio:

$$\frac{M}{G}=\frac{\overline{A}(\sigma_B^2+\overline{B}^2)-\overline{B}(\sigma_A\sigma_B p+\overline{A}\,\overline{B})}{\overline{B}(\sigma_B^2+\overline{A}^2)-\overline{A}(\sigma_A\sigma_B p+\overline{A}\,\overline{B})} \tag{13}$$

(The solutions for M and G are the numerator and the denominator of (13), respectively, divided by $(\sigma^2_A+\overline{A}^2)(\sigma^2_B+\overline{B}^2)-(\sigma_A\sigma_B p+\overline{A}\,\overline{B})^2>0$). It is shown in the Appendix that $\sigma^2_A>\sigma^2_B$ and that M>G. It could not be otherwise given that according to (7), ΔY_U is more responsive to ΔM than to ΔG. Our standard policy conclusion as to the appropriateness of monetary policy vis a vis changes in G and t, is once again confirmed. Our advice to the authorities would be now: «Do whatever changes in money supply and taxation are deemed appropriate with respect to the official economy, and let the interest rate take care of matters in the underground economy, aided by transaction costs considerations».

Such a laissez-faire type of advice seems to contradict the policymaking exercise that led to (13). For us, it was indeed only an exercise to help us verify the role of money from still another perspective. It is this role that should dictate policy and not a particular government wish about the size of the underground economy. After all, this economy acts counter cyclically and its output adds to official output, thus enhancing welfare. These two advantages of the underground economy are related with money, which simultaneously is decisive in controlling the size of this economy .

5. Conclusion

The emphasis we place on interest rate variations, echoes that placed originally by Poole (1970) as to the proper means of confronting uncertain spending via monetary policy. In the presence of underground activities, spending is indeed uncertain. Yet, in contrast with this standard policy prescription which advocates constancy of the money supply too, here interest rate variations are connected with money supply variations, because we do not view the official and the underground economy as one single entity. The focus is only on the underground economy and hence, we have no grounds for making any policy recommendations about the official economy. What we do say is just that money supply changes affect the underground economy and that from the viewpoint of controlling the expansion of this economy, it is good these changes to be moderated via interest rate changes as well. Moderation of the expansion of money contributes per se to this control, because it works against the financing of underground transaction costs, of the costs needed to keep these transactions secret .

The extent to which such policy considerations can be useful in practice depends on a variety of microeconomic factors that are intimately related with social circumstances. The relations can be impressively complex as the accounts of underground phenomena by e.g. Thomas (1992) and Williams and Windebank (1995) indicate. The socioeconomic inertia that appears to exist may not only incapacitate policy but also create for a government serious political problems if the authorities insist on policy implementation.

However, one advantage of the policy considerations advanced in this paper, is that they are not designed especially for the purpose of controlling the underground economy. In fact, there is no issue of design at all, since what we have done is to draw attention to how the two major policy instruments, money and public expenditure, influence underground activities. But, any deliberate use of these instruments against the underground economy needs not be publically announced as such; there can always be a good pretext within the context of the official economy .

APPENDIX

From (13), M/G>1 implies that

$$\frac{A}{B} > \frac{(\sigma_A^2 + \overline{A}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})}{(\sigma_A^2 + \overline{B}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})} \tag{A.1}$$

But,

$$\frac{(\sigma_A^2 + \overline{A}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})}{(\sigma_A^2 + \overline{B}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})} > \frac{(\sigma_A \sigma_B p + \overline{A}\ \overline{B})}{\sigma_B^2 + \overline{B}^2} \tag{A.2}$$

or

$$(\sigma^2_A + \overline{A}^2)(\sigma^2_B + \overline{B}^2) \ > \ (\sigma_A \sigma_B p + \overline{A}\ \overline{B})^2$$

which is true. The left-hand side of (A.2) is equal to the ratio of the right-hand side to which $(\sigma^2_A + \overline{A}^2)$ has been added to the numerator and $(\sigma_A \sigma_B p + \overline{A}\ \overline{B})$ has been added to the denominator. Since inequality (A.2) is true, what has been added to the numerator must be larger than what has been added to the denominator, i.e. $\sigma_A \sigma_B p + \overline{A}\ \overline{B} < \sigma^2_A + \overline{A}^2$ or

$$\sigma_A(\sigma_B p - \sigma_A) < \overline{A}(\overline{A}\ \overline{B}) \tag{A.3}$$

Also, note that
$$\frac{\sigma_A^2 + \overline{A}^2}{\sigma_A \sigma_B + \overline{A}\ \overline{B}} > \frac{(\sigma_A^2 + \overline{A}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})}{(\sigma_A^2 + \overline{B}^2) + (\sigma_A \sigma_B p + \overline{A}\ \overline{B})} \tag{A.4}$$

or $(\sigma^2_A + \overline{A}^2)(\sigma^2_B + \overline{B}^2) \ > \ (\sigma_A \sigma_B p + \overline{A}\ \overline{B})^2$

which is true. Again, the ratio of the right-hand side of (A.4) is equal to the ratio of the left-hand side to which $(\sigma_A \sigma_B p + \overline{A}\ \overline{B})$ and $(\sigma^2_B + \overline{B}^2)$ have been added to the numerator and to the denominator, respectively. The truth of (A.4) implies that what has been added to the numerator is less than the amount added to the denominator, i.e. $\sigma_A \sigma_B p + \overline{A}\ \overline{B} \ < \ \sigma^2_B + \overline{B}^2$ or

$$\overline{B}(\overline{A} - \overline{B})\, \sigma_B(\sigma_B - \sigma_A p) \tag{A.5}$$

From (A.3) and (A.5), we conclude that

$$\frac{\sigma_A}{A}(\sigma_A - \sigma_B p) > \frac{\sigma_B}{B}(\sigma_A p - \sigma_B) \Rightarrow \frac{\sigma_A^2 - \sigma_A - \sigma_B p}{\sigma_A \sigma_B p - \sigma_B^2} > \frac{\overline{A}}{\overline{B}} \qquad \text{(A.6)}$$

$\overline{A}/\overline{B} > 0$ means in turn that $(\sigma^2_A - \sigma_A\sigma_B p)/(\sigma_A\sigma_B p - \sigma_B^2) > 0$ and hence, that $\sigma^2_A > \sigma_A\sigma_B p$ and $\sigma_A\sigma_B p > \sigma^2_B$

which lead to the relationships

$$\sigma^2_A > \sigma^2_B,\ \sigma_A > \sigma_B p, \quad \text{and } \sigma_A p > \sigma_B \qquad \text{(A.7)}$$

Next, note that (A.1) has not been proven; we simply utilized the right-hand side of that inequality in order to derive (A.7). to prove that M/G>1, observe that

$$\frac{M}{G} > 1 \Rightarrow \frac{M}{G} = \frac{(\overline{A}/\overline{B})(\sigma_B^2 + \overline{B}^2) - (\sigma_A\sigma_B p + \overline{A}\,\overline{B})}{(\sigma_B^2 + \overline{A}^2) - (\overline{A}/\overline{B})(\sigma_A\sigma_B p + \overline{A}\,\overline{B})} > 1$$

$$\text{or } \frac{\overline{A}}{\overline{B}}(\sigma_B^2 + \overline{B}^2) - (\sigma_A\sigma_B p + \overline{A}\,\overline{B}) > \frac{\overline{A}}{\overline{B}}\frac{\overline{B}}{\overline{A}}(\sigma_A^2 + \overline{A}^2) - \frac{\overline{A}}{\overline{B}}(\sigma_A\sigma_B p + \overline{A}\,\overline{B})$$

$$\text{or } \frac{\overline{A}}{\overline{B}}\{(\sigma_B^2 + \overline{B}^2) - \frac{\overline{B}}{\overline{A}}(\sigma_A^2 + \overline{A}^2)\} > (\sigma_A\sigma_B p + \overline{A}\,\overline{B})(1 - \frac{\overline{A}}{\overline{B}})$$

$$\text{or } \overline{A}(\sigma_B^2 + \overline{B}^2) - \overline{B}(\sigma_A^2 + \overline{A}^2) > (\sigma_A\sigma_B p + \overline{A}\,\overline{B})(\overline{B} - \overline{A})$$

$$\text{or } \frac{\overline{A}\sigma_B^2 - \overline{B}\sigma_A^2 + \overline{A}\,\overline{B}(\overline{B} - \overline{A})}{(\overline{B} - \overline{A})} > \sigma_A\sigma_B p + \overline{A}\,\overline{B}$$

$$\text{or } \frac{\overline{A}\sigma_B^2 - \overline{B}\sigma_A^2}{\overline{B} - \overline{A}} > \sigma_A\sigma_B p \qquad \text{(A.8)}$$

If $(\overline{A}\sigma_B^2 - \overline{B}\sigma_A^2)/(\overline{B} - \overline{A}) \quad /\sigma_A^2$ is true, then (A.8) and hence, M/G>1, will be true, because by virtue of (A.7), $\sigma^2_A > \sigma_A\sigma_B p$. Indeed,

$$\frac{\overline{A}\sigma_B^2 - \overline{B}\sigma_A^2}{\overline{B}-\overline{A}} > \sigma_A^2 \Rightarrow \frac{\overline{A}\sigma_B^2}{\sigma_A^2} - \frac{\overline{B}\sigma_A^2}{\sigma_A^2} > \overline{B}-\overline{A} \Rightarrow \frac{\overline{A}\sigma_B^2}{\sigma_A^2} > 2\overline{B}-\overline{A}$$

which is true under ordinary circumstances, since $\overline{A}\sigma_B^2 / \sigma_A^2 > 0$ and $2\overline{B}-\overline{A} < 0$. Note that A>B means that $\overline{A} > \overline{B}$ or (e+j)gY+vs>sm and as soon as Y is in terms of million, billions or trillions of monetary units, it takes in practice an infinite interest elasticity of money, (as captured by m), for (e+j)gY+vs not to exceed sm by many, and not just two, times.

Notes

1. The literature on underground economy and tax evasion has stressed repeatedly that the two phenomena are two different things while Tanzi (1982) and Soldatos (1994) are among those who have emphasized that there can be underground economy without tax evasion.

2. Another complementary explanation of this connection between official interest rate and underground investment is offered later in section 3.

3. This was first observed by Gutmann (1977) and does not, of course, imply that the large use of cash pinpoints to the presence of a vivid underground economy. Philip (1949), however, had already suggested that one should add to the Keynesian motives for holding money, the motive of tax evasion. See also note 6.

4. This point is clarified below in the text and in notes.

5. The references here are Moore (1989) and Palley (1994, 1996).

6. There are many references here, but we draw attention to Goldfeld's finding that households and businesses hold much less currency than that in circulation; this finding was simply published in 1976, i.e. a year ago before Gutmann's work, which connected missing money with underground economy. The reservations that have been expressed as to such a connection are only minor, and this is why an approach advanced toward the measurement of the size of underground economy, is the so - called "monetary approach".

7. The model we develop in this section could be formulated mathematically too, but this would add only text, nothing else. For the mathematics, the reader may consult the references of note 5.

8. Presumably, the monetary authority does not take into account the nexus between money and underground economy when it expands the monetary base.

9. There are certainly many other motives to go underground, reviewed by Thomas (1962), Williams and Windebank (1995), and others. Undoubtedly, however, tax evasion is a major and powerful motive.

10. When we use the term "know", we mean "know from standard macrotheory" and not that (6) is associated with the considerations of the previous section directly. According

to that section, Δi should be a function of the policy control reserve base rather than a function of M. The use of (6), however, suffices for out purposes her. See note 18.

11. In view of the fact that i and t are percentages and that Δi or Δt are decimals varying around 0.01 for Δi, and 0.05 for Δt, the use of so small in value g and s, may not be intuitive, and this is why we finally obtain a ΔY_U of only 0.89 units. Nevertheless, it is trends what we are interested in while the comparisons of figures that will be made in a little while, are based on the order of magnitudes.

12. More specifically, having replaced Δ's by d's (which designate derivatives), we divide (7) by dY and then we integrate with respect to dY.

13. These numbers are approximations and come from: $\Delta Y_U 0$ iff $85.4\Delta M+\Delta G>2167$ and $\Delta M+0.0117096\Delta G > 25.374707$, respectively, given that $\Delta Y=100$. Now, as soon as M and G have the same coefficients in (7) and (8) and $Y=10\Delta Y$, we should also have Y_U 0 iff $85.4M+G>21670$ and $M+0.0117096G>253.74707$. Yet, carrying out the calculations based actually on (8), we find e.g. that $85.4M+G>21605$. The difference between 21670 and 21605 might be attributed to rounding up decimals and to some constant of integration, since (8) comes from the integration of (7). This is the reason we see no difference in working numerically with (7) rather than (8). See also note 11.

14. One might additionally argue that as we move up the Laffer curve, the responsiveness of Y_U to t, increases after some point and may be one reason this curve changes later slope. The increase of s in the beginning may not be so powerful as to increase Y_U, because both tax rate and tax receipts are low and hence, G is either low, which means that Y_U has to do more with the money - interest nexus, of high but financed by money, which means again the same thing.

15. Within the Laffer curve context, increases in tax receipts stemming from increased g of efficiency against tax evasion, does not imply reduced tax rate and hence, a decline in the incentive for underground operations.

16. In our model, there is no reason for cycles in Y, since transaction costs exist only in the underground economy in the form of "secrecy" costs. We postulate the presence of such cycles arbitrarily or as the result of cycles, thus establishing from still another point of view the negative sign of the derivative dY_U/dY.

17. According to "Standard Mathematical Tables", ω should exceed the number 5.3, which implies that Ψ should be between zero and 0.009983, with the latter figure corresponding to ω=5.3.

18. Implicit behind (6) and (7), i.e. behind the cornerstone relations of our discussion, has been the assumption that money supply is exogenous. It is an assumption that is at stake with the considerations of the second section of this paper. The assumption $\Delta Ms=\Delta Ms$, however, were made for reasons of simplicity, since as we shall see currently, the introduction of endogenous money supply does not alter the nature of our findings. To determine now the interest rate, we have to equate the money demand function, $\Delta M_d = k\Delta Y+n\Delta Y_U-m\Delta i$, with the money supply function, $\Delta M_s=\Delta M_s-a\Delta Y-b\Delta Y_U+f\Delta i$, where a, b, and f are positive coefficients; (see for their sign Teigen, 1978). In this manner, one obtains

$\Delta i=[(k+a)\Delta Y+(n+b)\Delta Y_U-\Delta \overline{M}_s]/(f+m)$ (6) which when inserted back to (5) gives

$$Y_u = \frac{[(e+j)gY+sv]\Delta\bar{M}+[s(f+m)]\Delta G-\{(k+a)[(e+j)gY+sv]-s(f+m)[(e+j)(1-t)-1]\}\Delta Y}{Y(e+j)[(f+m)+(n+b)g]sv(n+b)}$$

As soon as the denominator of this expression is positive, the signs of the coefficients of $\Delta\bar{M}$ and ΔG are positive too, as in (7). Also the bracketed expression accompanying Y in the numerator, is expected to be positive in most cases, i.e.

$(e+j)\ (k+a)gY\ +\ s(f+m)\ +\ sv\ (k+a)>s(e+j)\ (1-t)\ (f+m)$

since $(e+j)\ (k+a)gY>s(e+j)\ (1-t)\ (f+m)$ or $(k+a)gY>s\ (1-t)\ (f+m)$ given that Y is too large to justify an inequality with the opposite direction. Therefore, we obtain $\Delta Y_U/\Delta Y<0$ as in (7), too. What would be of interest is whether these results would change were prices to be introduced into the analysis, but this lies beyond the scope of the present paper.

19. Note that throughout this paper, we have taken tax evasion to be a motive for entering the underground economy; we did not study the phenomenon of tax evasion per se and thus, our results should be interpreted with caution in so far as this phenomenon is concerned.

20. The last point that should be made quite parenthetically, is that either (7) of (8) could be used to measure the size of the underground economy. We think that this "by - product" of the work in hand, has some merit, but we refrain from utilizing it the same way we do not proceed to an analysis incorporating prices; both tasks comprise separate research agendas.

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