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ON THE N-PLAYER BARGAINING PROBLEM: THE UNIFORM DENSITY APPROACH

By

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Abstract

In an earlier paper we showed that the Nash Bargaining solution for 2-player demand games can be obtained as a non-cooperative solution if each player assumes a uniform density over the choices of his opponent. In this paper we investigate the possibility of recapturing the Harsanyi-Nash bargaining solution for N-player games through an extension of the uniform density argument. We show that for certain games a direct generalization is possible. Also, in all games, the Harsanyi-Nash solution is obtained if each player bargains separately with every other player and ascribes a uniform density over his opponent's choices. (JEL subject code: C7)

1. Introduction

In Glycopantis and Muir (1994) it was shown that the renowned Nash bargaining solution, (Nash (1950, 1953)), for 2-player demand games can be recaptured as a non-cooperative solution if each player assumes a uniform density for the choices of the other player and then demands the quantity which maximizes his expected surplus utility payoff. The intuitive justification of this approach is that the players apply the principle of insufficient reason (Luce and Raiffa (1957)). Namely in the absence of any information apart from the set of possibilities a natural way to proceed to reach a decision is by assuming all choices of one's opponent to be equally likely. It is also reasonable to assume equally likely choices if the calculations required to establish exactly how one's opponent will play are very involved and costly. The principle of insufficient

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reason is an idea from the area of bounded rationality, (Bimmore (1990), Kreps (1990), and Simon (1987)), which in general describes ways in which a rational choice should be made when the economic agents are constrained by the amount of the information available and their computational abilities.

In an N-player demand game the players, PI, ..., PN, announce simultaneously and indepedently demands, in terms of utility, xi, ..., XN, respectively, which are greater or equal to the corresponding status-quo payoffs, *ξι, ..., ξπ-Ií* the vector of demands, (xi, ..., x_N), lies in the set of feasible payoffs, 5, which is assumed to be convex and compact, then every player receives what he asked for. If (xi,..., X_N) *φ* 2 then the demands are incompatible and the players receive their status-quo payoffs. The Nash equilibria are the set of the Pareto efficient vectors (xi, ..., XN) with (χι, ..., χ_N) > *(ξι, ..., ξϰ)* which, apart from very extreme cases, contains uncountably many points. Therefore the concept of Nash equilibrium, which is appropriate for non-cooperative games, leads to indeterminancy rather than to a unique solution of the demand game. On the other hand, it gives the set of payoffs over which the N players will negotiate. For 2-player games Nash (1950, 1953) gave arguments why a particular Pareto efficient utility payoff vector, the Nash bargaining solution which can be calculated easily, will be chosen by rational players as the outcome of the cooperative demand game. In Glycopantis and Muir (1994) the Nash bargaining solution was reviewed briefly, as well as certain approaches in the literature which have been employed to justify the Nash bargaining solution through non-cooperative games, and then what can be called the uniform density approach was introduced.

In the present paper we investigate the possibility of obtaining the analogue to the Nash bargaining solution for demand games with N players by extending in an appropriate way to such games the uniform density approach. The assumption is now that each player, in the absence of any specific information about the behaviour of the other players or if the calculations required in order to establish precisely how they will act are very involved and costly, will ascribe to each of his opponents, either simultaneously and independently to all of them or to one at a time, a uniform density over his choices.

Harsanyi (1977) has investigated bargaining games with N players. He defines a multilateral bargaining equilibrium to be one that implies bilateral bargaining equilibrium between any two players. He shows that such a solution can also be obtained from the Nash axiomatic approach when the postulates are taken to apply to N-player bargaining games.

As shown by Harsanyi (1977), the N-player bargaining game has as solution that of

Problem 1
Maximize $x_1x_2...x_N$
Subject to

$$g(x_1, x_2, ..., x_N) \leq 0$$
$$x_1, x_2, ..., x_N \geq 0$$

where Xj is the utility payoff to Pi and g (χι, X2,..., X_N) = O is the boundary of the convex set of feasible payoffs. The status-quo payoffs, following a normalization of the utility functions of the players, are taken throughout this paper to be equal to zero.

We shall refer to the solution to Problem 1 as the N-player Nash bargaining solution or the Harsanyi-Nash bargaining solution. We consider here the possibility of recapturing the Harsanyi-Nash bargaining solution by extending the uniform density argument employed in Glycopantis and Muir (1994).

A number of economic problems will lead naturally to the formulation of N-player demand games. As in the case of 2-player games, they are mainly in the areas of labour and industrial economics. They could refer, for example, to bargaining over wages and employment in a model with one employer and two unions, with status-quo payoffs the minimum obtainable profit and utilities, or to N oligopolists which form a cartel and wish to divide the resulting monopolistic profits, with status-quo payoffs the profits corresponding to the Cournot-Nash non-cooperative equilibrium.

For the symbolic problem of dividing a cake among N players the idea applied in Glycopantis and Muir (1994) can be generalized in the following sense. We assume that in making their calculations each player assigns an independent uniform distribution to the choices of each of the N-1 other players and that he chooses his own demand to maximize his expected payoff. We now show that the result of this type of decision is the N-player Nash bargaining solution.

First the Harsanyi-Nash bargaining solution is obtained from solving

Problem 2
Maximize $x_1x_2...x_N$
Subject to

$$x_1 + x_2 + ... + x_N \leq 1$$
$$x_1, x_2, ..., x_N \geq 0$$

which has answer $x_1^*, x_2^*, ..., x_N^* = 1/N$.

Consider now the direct generalization of our probabilistic argument which assumes that Pi assigns independent uniform distributions to the choices of all other players. This means that every other player is assumed to choose independently from [0, 1] with a uniform density d = 1 so that the joint density on $[0, 1]^{N-1}$ is again uniformly equal to 1.

If P1 chooses x_1, the probability that he will receive this payoff is the probability that, when the demands of all players are announced, $(x_1, x_2, ..., x_N)$ is in the feasible region, that is the probability that $x_1 + x_2 + ... + x_N \leq 1$. Since the joint density for $x_2, ..., x_N$ is 1, the probability is the (N-1)-volume of $\{(x_2, ..., x_N): x_2 + ... + x_N \leq 1-x_1$ and $x_i \geq 0\}$, namely $\int_0^{1-x_1} ... \int_0^{1-x_1-x_2-...-x_{N-2}} \int_0^{1-x_1-x_2-...-x_{N-1}} 1 \, dx_n dx_{n-1} ... dx_2$.

This integral can be calculated either directly, or through the argument concerning volumes given in Section 2 below. It is equal to $V(1-x_1)^{n-1}$ where $V>0$ is a factor independent of x_1. Therefore the expected return to P1 is $x_1 V(1-x_1)^{N-1} + 0\,[1-V(1-x_1)^{N-1}]$ since his status-quo payoff is 0.

It follows that P1 solves

Problem 3
Maximize $x_1 (1-x_1)^{N-1}$
Subject to $x_1 \in [0, 1]$

as V is irrelevant for the maximization.

This again gives as solution $x_1 = 1/N$ and since an idential argument can be made for all players we obtain again the Harsanyi-Nash bargaining solution.

Next we discuss a class of games for which a direct generalization of the probabilistic approach in Glycopantis and Muir (1994) gives the Harsanyi-Nash bargaining solution.

2. The Uniform Density Approach for N-Player Games.

Consider the N-player bargaining games given by $\mathfrak{X} = \{ (x_1, x_2, ..., x_N): \sum_{i=1}^{N} x_i^{\alpha_i} \leq 1$ and $x_i \geq 0 \}$ where $(\alpha_1, \alpha_2, ..., \alpha_N)$ are constants such that all $\alpha_i \geq 1$ so that the individual functions $x_i^{\alpha_i}$ are convex and the feasible set $\mathfrak{X}$ is convex. For example $\alpha_i = 1$ for all i gives the cake game and $\alpha_i = 2$ for all i gives a hypersphere game.

The Harsanyi-Nash solution of such a game is that of

Problem 4

Maximize $\sum_{i=1}^{N} \log x_i$

Subject to $(x_1, x_2, \ldots, x_N) \in \mathfrak{X}$

This has as Lagrangean function $L = \sum_{i=1}^{N} [\log x_i - \lambda (x_i^{\alpha_i} - (1/N))]$ and through routine calculations we obtain the solution $x_i{*} = [\beta_i / \sum_{j=1}^{N} \beta_j]^{\beta_i}$ where $\beta_i = 1/\alpha_i$.

Next we show that the probabilistic argument of assigning by each player, simultaneously and independently, a uniform density to the choices [0, 1] of each one of his opponents results in the same solution. Take the point of view of P1. He chooses some $x_1 \in [0, 1]$ and needs to know the probability of $(x_1, x_2, \ldots, x_N) \in \mathfrak{X}$ given this x_1, which is the (N-1)-volume of the region $\sum_{i=2}^{N} x_i^{\alpha_i} \leq 1 - x_1^{\alpha_1}$ in $\mathfrak{X}$, since the joint uniform density assigned by P1 to the joint choices of P2, P3, ..., PN is 1.

We insert a short digression on volumes. Suppose we have an object O in R^N. Its volume is given by $\mathrm{Vol}(O) = \int_O 1 dx_1 dx_2 \ldots dx_N$. Suppose now that two objects O and O′ are related so that $(x_1, x_2, \ldots, x_N) \in O'$ iff $(x_1/\mu_1, x_2/\mu_2, \ldots, x_N/\mu_N) \in O$ for $\mu_1, \mu_2, \ldots, \mu_N > 0$. Then

$$\mathrm{Vol}\,(O') = \mu_1\mu_2\ldots\mu_M \,\mathrm{Vol}\,(O). \tag{1}$$

This is shown as follows. Take $\mathrm{Vol}(O') = \int_{O'} 1 dx_1 dx_2 \ldots dx_N$ and introduce change of variables $y_i = x_i/\mu_i$. Then $\mathrm{Vol}(O') = \int_O \mu_1\mu_2\ldots\mu_N \, dy_1 dy_2 \ldots dy_N = \mu_1\mu_2\ldots\mu_N$ Vol(O) which proves the result.

Now we return to the N-player games we are considering and apply the result immediately above. Let the region $O = \{ (x_2, \ldots, x_N): \sum_{i=2}^{N} x_i^{\alpha_i} \leq 1 \text{ and } x_i \geq 0\}$ have volume V and consider $O' = \{ (x_2, \ldots, x_N): \sum_{i=2}^{N} x_i^{\alpha_i} \leq A \text{ and } x_i \geq 0 \}$. This region is of course identical to that given by $\{(x_2, \ldots, x_N): \sum_{i=2}^{N} (x_i / A^{\beta_i})^{\alpha_i} \leq 1 \text{ and } x_i \geq 0\}$, where $\beta_i = 1/\alpha_i$, and employing the result in (1) with $\mu_i = A^{\beta_i}$, i = 2, ..., N, we obtain $\mathrm{Vol}(O') = A^{(\sum_{i=2}^{N} \beta_i)} V$. Therefore the probability of $(x_1, x_2, ..., x_N) \in \mathfrak{X}$, given x_1, is $\mathrm{Vol}(O')$ with $A = 1-x_1^{\alpha_1}$, since the joint uniform density assigned by P1 to the joint choices of the other players is 1, so the expected return to P1 from choosing x_1 is $x_1 (1 - x_1^{\alpha_1})^{(\sum_{i=2}^{N} \beta_i)} V$.

Obviously in the problem of dividing a cake among N players all α_i's and β_i's are equal to 1 and the (N - 1)-volume of the feasible set $\{(x_2, ..., x_N): x_2 + ... + x_N \leq 1-x_1 \text{ and } x_i \geq 0\}$ is $V(1 - x_1)^{N-1}$ as stated in Section 1, where V is volume of $\{(x_2, ..., x_N): x_2 + ... + x_N \leq 1 \text{ and } x_i \geq 0\}$.

Since in the games above V is merely a numerical factor independent of x_1, P1 must obtain the solution of

Problem 5

Maximize $x_1 (1 - x_1^{\alpha_1})^{(\sum_{i=2}^{N} \beta_i)}$

$x_1 \in [0, 1]$

which on routine calculations is shown to have solution $x_1 = [\beta_1 / \sum_{j=1}^{N} \beta_j]^{\beta_1} < 1$. An identical argument can be made for all players and we obtain again the Harsanyi-Nash bargaining solution $x_i^* = [\beta_i / \sum_{j=1}^{N} \beta_j]^{\beta_i}$.

By an analogous argument the class of N-player games for which the Harsanyi-Nash bargaining solution can be justified through the uniform density approach can be extended to $\mathfrak{X} = \{ (x_1, x_2, ..., x_N): \sum_{i=1}^{N} c_i x_i^{\alpha_i} \leq c \text{ and } x_i \geq 0\}$ where all c_i's and c are positive constants and $\alpha_i \geq 1$ for all i.

However the probabilistic approach described above and the Harsanyi-Nash solution do not always match up as the following example shows.

We consider the 3-player demand game the feasible payoffs set of which is given by the expression $-\log(1-x_1) - \log(1-x_2) - \log(1-x_3) \leq \log e = 1$ with $0 \leq x_1, x_2, x_3 \leq 1-e^{-1}$.

Alternatively, we can write the feasible set as $(1-x_1)^{-1}(1-x_2)^{-1}(1-x_3)^{-1} \leq e$ with $0 \leq x_1, x_2, x_3 \leq 1-e^{-1}$. Therefore the equation of the boundary of the feasible payoffs set can be given in the implicit form $g(x_1, x_2, x_3) = (1-x_1)^{-1}(1-x_2)^{-1}(1-x_3)^{-1} - e = 0$ from which we obtain $x_1 = f^1(x_2, x_3) = 1 - e^{-1}(1-x_2)^{-1}(1-x_3)^{-1}$, $x_2 = f^2(x_1, x_3) = 1 - e^{-1}(1-x_1)^{-1}(1-x_3)^{-1}$, and $x_3 = f^3(x_1, x_2) = 1 - e^{-1}(1-x_1)^{-1}(1-x_2)^{-1}$.

We prove the convexity of the feasible set, $\mathfrak{X}$, or equivalently the concavity of the boundary, Pareto efficient, surface $x_3 = f^3(x_1, x_2) = 1 - e^{-1}(1-x_1)^{-1}(1-x_2)^{-1}$ as follows.

Consider (x_1', x_2', x_3') and $(x_1'', x_2'', x_3'') \in \mathfrak{X}$. Since $-\log(1-x)$ is strictly convex, we have, for $0 \leq \lambda \leq 1$,

$$-\log(1 - (\lambda x_1' + (1-\lambda)x_1'')) - \log(1 - (\lambda x_2' + (1-\lambda)x_2'')) - \log(1 - (\lambda x_3' + (1-\lambda)x_3'')) \leq$$
$$- \lambda\log(1-x_1') - (1-\lambda)\log(1-x_1'') - \lambda\log(1-x_2') - (1-\lambda)\log(1-x_2'') - \lambda\log(1-x_3') -$$
$$(1-\lambda)\log(1-x_3'') \leq \log e. \tag{2}$$

Therefore $(\lambda x_1' + (1-\lambda)x_1'')$, $(\lambda x_2' + (1-\lambda)x_2'')$, $(\lambda x_3' + (1-\lambda)x_3'') \in \mathfrak{X}$. hence $\mathfrak{X}$ is convex and the boundary surface is concave.

First we obtain, for the game above the Harsanyi-Nash bargaining solution. It is the solution of

Problem 6
Maximize $x_1x_2x_3$
Subject to

$$(1 - x_1)^{-1}(1 - x_2)^{-1}(1 - x_3)^{-1} \leq e$$
$$0 \leq x_1, x_2, x_3 \leq 1 - e^{-1}.$$

In view of the symmetry of both the objective function and the constraint in all the variables we get $x_1^* = x_2^* = x_3^* = 1 - e^{-1/3}$.

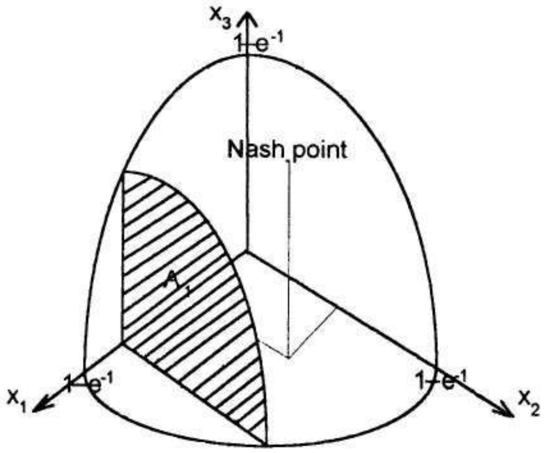
Next we calculate the demands of the players following the uniform density approach for 3 players. We consider P1. He ascribes over the choices, $[0, 1 - e^{-1}]$, of each of the other players the uniform density $e/(e - 1)$. Therefore he ascribes uniform density $e^2 / (e - 1)^2$ to the choice pairs in $[0, 1-e^{-1}]^2$.

Now if P1 decides to demand x_1 he will receive x_1 with probability $m_1(x_1)$, which is the measure of the feasible subset $S_1(x_1) = \{ (x_2, x_3): g(x_1, x_2, x_3) \leq 0 \}$, and 0, his status-quo payoff, with probability $1 - m_1(x_1)$. Therefore he wants to choose x_1 to maximize his expected peyoff $E_1(x_1) = x_1m_1(x_1)$.

In view of the uniform density fact we write

$$E_1(x_1) = [e^2 / (e-1)^2]\, x_1\, A_1(x_1) \tag{3}$$

where $A_1(x_1)$ denotes the area of $S_1(x_1)$.



Graph 1

In the graph above A_1 is the number that is assigned to the shaded set, obtained from the feasible region for a specific x_1. The picture helps to explain the calculations below.

Following the graph we see that, given x_1, we want to integrate $x_3 = f^3(x_1, x_2)$ over the interval $[0, x_2 = f^2(x_1, 0)]$. Therefore we have

$$A_1(x_1) = \int_0^{1-e^{-1}(1-x_1)^{-1}} \{1-e^{-1}(1-x_2)^{-1}(1-x_1)^{-1}\}dx_2 = [x_2 + e^{-1}(1-x_1)^{-1}\log(1-x_2)]_0^{1-e^{-1}(1-x_1)^{-1}} \tag{4}$$

Performing the integration in (4) we obtain

$$A_1(x_1) = 1-e^{-1}(1-x_1)^{-1} - e^{-1}(1-x_1)^{-1}(\log(1-x_1)+1) \tag{5}$$

and employing (5) we wish to maximize the expected payoff to P1, given by

$$E_1(x_1) = e^2(e-1)^{-2}x_1 A_1(x_1) = e^2(e-1)^{-2}(x_1-2x_1e^{-1}(1-x_1)^{-1} - x_1e^{-1}(1-x_1)^{-1}\log(1-x_1)) \tag{6}$$

over $[0, 1-e^{-1}]$.

Equivalently we wish to maximize

$$K(x_1) = ex_1 - 2x_1(1-x_1)^{-1} - x_1(1-x_1)^{-1}\log(1-x_1). \tag{7}$$

Calculating the derivatives we obtain

$$K'(x_1) = e - 2(1-x_1)^{-1} - x_1(1-x_1)^{-2} - (1-x_1)^{-1} \log(1-x_1) - x_1(1-x_1)^{-2} \log(1-x_1) \quad (8)$$

and

$$K''(x_1) = - 2(1-x_1)^{-2} - x_1(1-x_1)^{-3} - 2(1-x_1)^{-2} \log(1-x_1) - 2x_1(1-x_1)^{-3} \log(1-x_1). \quad (9)$$

Multiplying $K''(x_1)$ by $(1-x_1)^3$ we see that in $[0, 1-e^{-1}]$ it has the same sign as $G(x_1) = -2 + x_1 - 2\log(1-x_1)$, which is originally negative and then positive. Hence the function $K(x_1)$ is originally concave and then convex. Since $K'(1-e^{-1/3}) > 0$ and $K''(1-e^{-1/3}) < 0$, at the point $1-e^{-1/3}$ the function $K(x_1)$ is still concave and rising and therefore its maximum occurs at a value greater than $1-e^{-1/3}$.

Now because of the symmetry of the players' problems each demands a quantity larger than $1-e^{-1/3}$, his share in the Harsanyi-Nash bargaining solution, and the vector of demands lies outside the set of feasible payoffs. Therefore the players end up with their status-quo payoff which is 0.

The above example shows that a direct generalization of the uniform density approach for the 2-player games to the N-player games will, in general, not confirm the Harsanyi-Nash bargaining solution. On the other hand, as shown below, a generalization, based on a bounded rationality approach, is obtained in a pairwise decentralization of the game.

3. A Generalization of the Probabilistic Approach

The Harsanyi-Nash bargaining solution can be obtained through the probabilistic (uniform density) approach, by allowing any pair to calculate first their demands conditional on the N-2 players having fixed quantities allocated to them. The assumption is now that each player is only able to handle 2-player games and makes all possible calculations before he announces his demand. There is an affinity here between this approach and the requirement by Harsanyi that a multilateral bargaining equilibrium should be such that it implies bilateral equilibrium between any two players.

As in Glycopantis and Muir (1994), the principle of insufficient reason, which falls within the area of bounded rationality, operates when, in the pairwise decentralized games, each player assigns a uniform density over the possible choices of the remaining player whose allocation is also not fixed. The rationality of the players is also bounded in assuming that they can only do their calculations when they are playing against only one more player.

Each player, say Pi, reasons completely separately. He knows that he must announce a demand, which is a number, and that his bounded rationality allows him to handle only 2-player games. He assumes that N-2 players take definite amounts and he is then left to play a 2-player game with the remaining player, say Pj.

The fact that N-2 demands are fixed in $g(x_1, x_2, ..., x_i, ..., x_j, ..., x_N) \leq 0$ implies that Pi and Pj play a game with negotation set, say, $G(x_i, x_j) \leq 0$. Pi assumes a uniform density over the choices of Pj, that is over $[0, \tilde{x_j}]$ where $\tilde{x_j}$ solves $G(0, x_j) = 0$. In relation to the original game this is a uniform density conditional on the assumed N-2 quantities.

Maximizing his expected payoff as in Glycopantis and Muir (1994), Pi calculates his demand as a function of the N-2 quantities assumed to be fixed. We call this a reaction function and its derivation is part of the thought process that leads to the calculation of the demand that the player will announce.

Since there are N-1 ways of choosing the N-2 players, or, equivalently, the remaining player, Pi calculates N-1 reaction functions. Each reaction function can be placed on the boundary of $\mathfrak{X}$, calculating the missing coordinate from $g(x_1, x_2, ..., x_N) = 0$ by inserting the values of the N-2 variables and the implied value for x_i.

As it will be argued below, the reaction functions of Pi intersect on the boundary of $\mathfrak{X}$ at a single point $(x_1^{i*}, x_2^{i*}, ..., x_N^{i*})$. This is the only point that has the property that, given any N-2 of these demands, playing the game with the remaining player implies x_i^{i*}.

From the point of view of Pi, the announcement X_i^{1*} is the only demand for which there exists an efficient vector such that $x^{1'}$ is the outcome of the uniform density approach to the pairwise games which form when the demands of any of the remaining players are given. It is therefore only rational that Pi will announce x_i^{1*} as any other announcement will imply that, no matter what the other players have demanded, in at least one pairwise game he should have asked for a different quantity.

We shall now show that the reaction functions of Pi on the boundary of 5 intersect at a single point. Indeed we shall show that they intersect at the unique

Harsanyi-Nash point $(x_1^*, x_2^*, ..., x_N^*)$. It follows, since each Pi reasons in the same manner, that each player will demand his Harsanyi-Nash allocation.

Consider, for example, P1 and assume that the demands of P3, P4, ..., PN are fixed at $x_3', x_4', ..., x_N'$ respectively. He is imagining that he is playing a game with P2, with feasible payoff region $g(x_1, x_2, x_3', x_4', ..., x_N') \leq 0$ and status quo payoffs (0, 0).

Let $g(x_1, x_2, x_3', x_4', ..., x_N') = 0$ imply $x_2 = f^2(x_1, x_3', ..., x_N')$. P1 adopts the uniform density approach and calculates x_1 which solves

Problem 7
Maximize $x_1 f^2(x_1, x_3', ..., x_N') / d$
Subject to

$$x_1 \in [0, \tilde{x_1}]$$

where $\tilde{x_1}$ solves $g(x_1, 0, x_3', ..., x_N') = 0$ and $d = f^2(0, x_3', ..., x_N')$ gives P2's maximim possible demand in the conditional game.

From the problem above we obtain the reaction function $x_1 = X_1^2(x_3', ..., x_N')$ where the superscript denotes the player with whom P1 is negotiating. This reaction function is then placed on the boundary of $\mathfrak{X}$ by calculating x_2 from

$$g(X_1^2(x_3', ..., x_N'), x_2, x_3', ..., x_N') = 0 \tag{10}$$

On the boundary of $\mathfrak{X}$, the reaction function traces Nash bargaining solutions for games conditional on $(x_3', ..., x_N')$. This follows from the fact (see also Glycopantis and Muir (1994)) that P1's demand is the same as the x_1 obtained from the solution of

Problem 8
Maximize $x_1 x_2$
Subjecto to

$$x_2 \leq f^2(x_1, x_3', ..., x_N')$$
$$x_1 \in [0, \tilde{x_1}]$$

or equivalenty

Problem 9
Maximize $x_1 x_2$
Subject to

$$g(x_1, x_2, x_3', ..., x_N') \leq 0$$
$$x_1, x_2 \geq 0$$

and from the fact the Nash bargaining solution is always on the boundary of the feasible payoff region.

It follows that when $x_1 = X_1^2(x_3', ..., x_N')$ is placed, through (10), on the boundary of $\mathfrak{X}$ it goes through the unique Harsanyi-Nash bargaining solution $(x_1^*, x_2^*, ..., x_N^*)$, and so do all such reaction functions of P1, as the Harsanyi-

Nash solution implies the Nash bargaining solution for all pairwise decentralized games.

The question arises whether they also intersect at any other point. However this is impossible because at any such point we would have had the Nash bargaining solution for all pairwise games involving P1, and this implies that the necessary, first order, conditions for the unique Harsanyi-Nash solution are also satisfied.

Therefore we have obtained a generalization of the probabilistic approach, based on a bounded rationality argument, which offers a justification of the Harsanyi-Nash bargaining solution.

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