«ΣΠΟΥΔΑΙ», Τόμος 47, Τεύχος 1ο-2ο, Πανεπιστήμιο Πειραιώς / «SPOUDAI», Vol. 47, No 1-2, University of Piraeus

A BINOMIAL RANDOM SUM OF PRESENT VALUE MODELS IN INVESTMENT ANALYSIS

By

Aggeliki Voudouri
University of Athens

Evangelos Ntziachristos
University of Athens

Abstract

Stochastic present value models have been widely adopted in financial theory and practice and play a very important role in capital budgeting and profit planning. The purpose of this paper is to introduce a binomial random sum of stochastic present value models and offer an application in investment analysis. (JEL:C51)

1. Introduction

Prudent planning for scheduled future financial commitments has become a crucial element of investment strategy and consequently stochastic present value models are of great importance in theoretical economics. The analysis of stochastic present value models provides one of the more powerful techniques applicable to financial economics. These models rely on the results of probability theory and aim to provide management with sufficient information for financial decision making under conditions of uncertainty. This paper proposes a random sum of stochastic present value models which can be shown to have interesting applications in financial economics. More precisely the paper introduces a binomial random sum of present values of single random cash flows under exponentially distributed timings. The paper also provides an application of this random sum in investment analysis. From a practical point of view the paper extends the applications in financial economics of a wide class of probability distributions. Properties of this class of distributions have been established by Olshen and Savage (1970), Dharmadhikari and Jogdeo (1974), Gomes and Pestana (1978), Artikis (1981), Artikis (1982), Alamatsaz (1985), Artikis (1987), Artikis et al (1988), Artikis arid Voudouri (1987).

The main part of the paper consists of three sections. Section two is devoted to a present value model of a single random cash flow under exponential timing and section three introduces a binomial random sum of such present value models. Section four establishes an application of this random sum in investment analysis.

2. A Present Value Model

Present value models play a very important role in financial theory and management. The literature has for the main part concentrated on purely deterministic models aimed at the construction of explicit present value formulae under conditions of certainty. Most of the stochastic treatments of present value models concentrate only on the establishment of mean value, variance and in some cases semivariance. Stochastic methods which fully exploit the properties and, under certain assumptions derive the distribution function of a present value model, are the most powerful tools of the theory of stochastic discounting, Rosenthal (1978). In general, any analytical determination of the distribution function of a stochastic present value model is extremely difficult. However establishing this distribution function as a member of a given class of distribution functions is of great importance. Recently, Artikis and Jerwood (1991) and Artikis et al (1993) have considered the application of two wide classes of distribution functions in certain present value models.

Let X represent a payment to be paid at some future time T. Considering continuous discounting of X when the force of interest is r, it can be shown that the present value of X is given by

$$V = Xe^{-rT} \tag{1}$$

Artikis and Jerwood (1991) considered a stochastic formulation of the present value model in (1) by assuming that r is constant and X, T are continuous nonnegative and independent random variables with X arbitrarily distributed and T exponentially distributed with parameter μ. Under these assumptions the characteristic function of V is given by

$$\phi_V(u) = \alpha\int_0^1 \phi_X(u\omega)\, \omega^{\alpha-1}\, d\omega \ , \tag{2}$$

where $\alpha = \mu/r$ and $\phi_X(u)$ is the characteristic function of X. Characteristic functions of the form (2) belong to α-unimodal distributions, Olshen and Savage (1970). From the fact that the exponential distribution is the most commonly encountered timing distribution it follows that the class of α-unimodal distributions is very important in present value models of the form (1). This paper is devoted to a binomial random sum of present value models of the form (1) when T is exponentially distributed.

3. Binomial Random Sums in Continuous Discounting

Let $\{ Y_n, n = 1, 2, \ldots\}$ denote a sequence of independent and identically distributed random variables with characteristic function $\phi_Y(u)$. Let N represent a nonnegative integer-valued random variable independent of $\{Y_n, n = 1, 2, \ldots,\}$ with probability generating function $P_N(z)$. The characteristic function of the random sum

$$S = Y_1 + Y_2 + \ldots + Y_N$$

is given by

$$\phi_S(u) = P_N(\phi_Y(u)).$$

Random sums are important as basic models in theory and practice, Feller (1966). If N is binomially distributed with probability generating function

$$P_N(z) = (pz + q)^n$$

where $0<p<1$, $p + q = 1$, then the random variable S is called a binomial random sum and the corresponding characteristic function has the form

$$\phi_S(u) = (p\phi_Y(u) + q)^n \tag{3}$$

Binomial random sums have interesting applications in insurance and risk management, Teugels (1985). The purpose of this section is to introduce a new binomial random sum of present value models.

Let X_k. $k = 1, 2, .., n$ be continuous independent nonnegative and identically distributed random variables with characteristic function $\phi_x(u)$ and let T_k, $k= 1, 2, \ldots, n$ be independent exponentially distributed, with parameter μ, random variables. We suppose that the random variables X_k, $k = 1, 2, \ldots n$, are independent of the random variables T_k, $k = 1, 2, \ldots, n$. The random variables

$$V_k = X_k e^{-rT_k}, \; k = 1, 2, \ldots, n$$

are continuous nonnegative and identically distributed with characteristic function given by (2), Artikis and Jerwood (1991). If N is a binomially distributed random variable with parameters n, p and independent of the random variables V_k, k = 1, 2, ..., n the then random variable

$$L = V_1 + V_2 + \dots + V_N$$

is a binomial random sum. From (2) and (3) we conclude that the characteristic function of the binomial random sum L is given by

$$\phi_L(u) = \left(p\alpha \int_0^1 \phi_x(u\omega)\, \omega^{\alpha-1}\, d\omega + q \right)^n .$$

Hence the random variable L is a binomial random sum of present value models of the form (1) with the random variable T exponentially distributed or equivalently a binomial random sum of α-unimodal random variables.

4. An Application of the Random Sum in Investment Analysis

Ar time 0 we consider n independent investment proposals. Let X_k, T_k, k = 1, 2, ..., n denote the salvage value and the duration respectively of the k-th investment proposal. Considering continuous discounting of X_k when the force of interest is r then

$$V_k = X_k e^{-rT_k}$$

is the present value of the salvage value X_k. We suppose that X_k, k = 1, 2, ..., n are continuous independent nonnegative and identically distributed random variables with characteristic function $\phi_x(u)$ and that T_k, k = 1, 2, ..., n are independent exponentially distributed, with parameter μ, random variables. Moreover, we suppose that the random variables X_k, k = 1, 2, ..., n are independent of the random variables T_k, k = 1, 2, ..., n. Let C_k represent the cost of the k-th investment proposal and suppose that C_k, k = 1, 2, ..., n are continuous independent nonnegative and identically distributed random variables with distribution function $F_C(c)$ and that C_k, k = 1, 2, ..., n are independent of X_k, k = 1, 2, ..., n and T_k, k = 1, 2, ..., n.

An acceptable investment proposal may be one whose cost is less than some critical value c_0 with probability

$$p = P[C_k < c_0] = F_C(c_0).$$

If the discrete random variable N denotes the number of the acceptable investment proposals at time 0 then N is binomially distributed with probability function

$$P[N = k] = \binom{n}{k} p^k q^{n-k}, \qquad k = 0, 1, \ldots, n$$

and independent of the random variables V_k, $k = 1, 2, \ldots, n$. Hence the binomial random sum

$$L = V_1 + V_2 + \ldots + V_N$$

denotes the sum of the present values of the salvage values of the acceptable investment proposals.

5. Concluding Remarks

The distribution function which corresponds to a binomial random sum of present value models is very complicated, but the possibility of applying numerical methods and approximations to this distribution function facilitates the use of such binomial random sums in investment analysis. Moreover Monte Carlo simulation techniques provide a viable means to verify, in some practical way, the consequences of the binomial random sum of present value models provided by the paper.

In general, any analytical determination of the distribution function of a binomial random sum of present value models is intrinsically difficult. However, it can be shown that this distribution function is related to certain known classes of distribution functions. In particular, if we suppose that the timings are exponentially distributed then the distribution function of a binomial random sum of present value models is related to a wide class of mixtures of probability distributions.

References

Alamatsaz, M. (1985) *Note on an Article by Artikis,* Acta Mathematicae Academiai Scientiarum Hungaricae 45, 159-162.

Artikis, T. (1981) *Properties of Certain Transformed Characteristic Functions,* SERDICA Bulgaricae Mathematicae Publicationes 7, 387-389.

Artikis, T. (1982) *A Note on Certain Power Mixtures,* Acta mathematicae Academiai Scientiarum Hungaricae 39, 69-72.

Artikis, T. (1987) *On Certain Classes of Distributions,* Colloquium Mathemancum LIII, 315-321.

Artikis, T., and Voudouri, A. (1987) *On the Present Value of Random Cash Flows Under Random Timing,* Bulletin of the Greek Mathematical Society 28, 57-63.

Artikis, T. Jerwood, D. and Voudouri, A. (1988) *An Integral Transformation for Characteristic Functions,* Zastosowania Matematyki Applicationes Mathematicae 20, 1 47-52.

Artikis, T. and Jerwood, D. (1991). *On the Unimodality of Present Value Distributions,* European Journal of Operational Research 52, 107-112.

Artikis, T., Voudouri, A. and Jerwood, D. (1993) *Analytical and Computer Simulation Techniques for a Stochastic Model Arising in Discounting Continuous Uniform Cash Flows,* Mathematical and Computer Modelling 18, 9-16.

Dharmadhikari, S. and Jogdeo, K. (1974) *Convolutions of α-unimodalDistributions,* Z. Wahrsch. Verw. 30, 203-208.

Feller, W. (1966) *An Introduction to Probability Theory and Its Applications 2,* John Wiley & Sons, New York.

Gomes, M., and Pestana, D. (1978) *The Use of Fractional Calculus in Probability,* Portugaliae Mathematica 37, 259-271.

Olshen, R., and Savage, L. (1970) *A Generalized Unimodality,* Journal of Applied Probability 7, 21-34.

Rosenthal, R. (1978) *The Variance of Present Worth of Cash Flows Under Uncertain Timing,* The Engineering Economist 23, 163-170.

Teugels, L. (1985) *Approximation and Estimation of Some Compound Distributions,* Insurance: Mathematics and Economics 4, 143-153.