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Investors' Behavior in Alternative Asset Classes

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Abstract

We investigate whether alternative asset classes should be included in optimal portfolios of the most prominent investor personae in the Behavioral Finance literature, namely, the Cumulative Prospect Theory, the Markowitz and the Loss Averse types of investors. We develop a stochastic spanning approach for each type of investor. Using the Stochastic Spanning criterion, we construct optimal portfolios with and without alternative assets, namely FX, Commodities, Real Estate and precious metals. Our out of sample comparative performance analysis indicates that investors impression of gains and losses affects significantly the composition and aggregate performance of optimal portfolios and that the alternative asset classes examined are attractive attracted under risk conditions.

JEL Classification: C12, C13, C15, C44, D81, G11, G14.

Keywords: Parametric and Non-parametric tests, Second Order Stochastic Dominance, Stochastic Spanning, Cumulative Prospect Theory, Loss Aversion, Markowitz Theory, Probability Weighting.

1. Introduction

Traditionally, U.S. investors focus on all stocks and bonds portfolios. The optimal portion in either asset class, and a sufficient number of individual items, ensures diversification and reducing the idiosyncratic risk. However, the financialization of alternative asset classes, e.g., commodities and real estate, and the possibility to participate in international markets, allows investors to benefit from different return-risk characteristics and especially low correlation of the alternative assets with U.S. stocks or bonds. Moreover, there is empirical as well as experimental evidence that investors are not always globally risk averse, but instead they seem to exhibit local risk-seeking behavior (i.e., the utility function has convex segments). The aim of this paper is to analyze investor preferences and beliefs by testing whether investors with more complex attitudes towards risk include alternative asset classes in their portfolios.

We construct two types of portfolios. The first consists of stock and bond indices while the second also includes commodities, foreign exchange (FX), real estate and precious metals. In this study, we extend the literature on asset class diversification and employ the stochastic dominance approach to test whether commodities, currencies, real estate and precious metals should be included in stock- bond portfolios to improve the investment universe investor. To do so, we employ six different investor types (personae) with respect to risk preferences, in order to capture various decision-making approaches, as well as financial behaviors.

These personae outline the following investor types: the one who is risk averse in gains and risk seeker in losses (investor with S-shaped utility functions), the investor who is risk lover in gains and risk averse in losses (investor with reverse S-shaped utility functions), the globally risk averse investor whose attitude towards risk serves as the benchmark, the so-called "naive" investor who employs the simple investing strategy (or heuristic) 1/N, where N is the number of assets included in a portfolio. Moreover, we employ the investor with indifferent (neutral) preferences towards risk (or uncertainty) and finally the investor with pronounced aversion to losses. The first type is the Cumulative Prospect Theory investor (CPT, Tversky and Kahneman 1992), while the second one is the Markowitz type investor (Markowitz 1952). All investor personas buy-and- hold the two aforementioned portfolio types, namely the "Traditional" and the "Enhanced" one. The "Traditional" (T) contains stocks and bonds. The "Enhanced" (E), is an enrichment of the Traditional enhanced each time with one of the four alternative asset classes. Thus, four different portfolios are constructed for each persona. In this way we conduct four core experiments with 6 sub-experiments each, covering all investor types for all asset classes.

We use these particular alternative asset classes because they have attracted investors' interest, since they have lower correlations with stocks and bonds. More particularly, real estate and precious metals were considered as safe havens and they were extensively used in hedging strategies against periods of financial turmoil. However, the financial crisis of 2008 revealed significant and deep pathogens in the real estate market and the majority of invested capital was drawn out, leaving the market full of unsuitable assets that could not be liquefied nor traded. On the other hand, the precious metals market has exhibited smoother ups and downs exhibiting moderate variability. Finally, commodities and FX are markets with high idiosyncratic risk because hundreds of affecting variables shape their prices while speculation, as well as arbitragers, find quite often a fertile environment for exercise, leaving these two markets susceptible to price manipulation.

Until recently, there are numerous studies on the application of CPT, each one taking a slightly different approach. These different approaches regard the employment of some or all elements of CPT such as, the curvature of the value function, the distortion of objective probabilities and loss aversion. However, almost all of them apply CPT on time series, usually for various assets, in order to extract the so-called CPT values that can be used in various contexts, for example on asset price prediction (Barberis et al. 2001). The main reason CPT is applied on time series is the availability of historical data, and by applying rolling window analysis, one can extract information about behavioral phenomena (e.g. overconfidence, i.e., excessive trading) that may affect prices or even propose a solution to observed but still unexplained existing phenomena, such as the Equity Premium Puzzle (Benartzi and Thaler 1995). Usually, these phenomena exhibit time dependence for the simple reason that they follow specific periodic patterns.

Nevertheless, one interesting question concerns not the time series but rather the cross-section of financial data, regarding how they can be exploited in order to provide concrete and meaningful behavioral insights. This is our motivation for the reason why we choose to apply CPT, Markowitz as well as the rest types of preferences/ attitudes, on the cross section of our dataset. If the cross-section is considerably "wide", we argue that the use of the aforementioned theoretical contexts can not only be justified but also reveal interesting outcomes. In this way, we can construct a "behavioral" dataset which can be used in order to "dictate" preferences, choices and thus the way a portfolio is going to be diversified. Basically, the three different personae observe the data from a different point of view and create a new auxiliary dataset, based on subjective perception. This procedure results into financial data analysis from a Behavioral Finance perspective. It is *as if* the impression of an investor, who is evaluating a dataset subjectively, is imprinted on a new dataset, namely; the behavioral one. This new dataset is the one that drives the investor's strategy because according to Behavioral Portfolio theory, it all boils down to how gains and losses are perceived from investors. This aforementioned distortion, how is it done, if it is done, when is it done and so on, formulates the main dispute between classical Finance scholars and behaviorists. Until today, behaviorists seem to pave the way due to pronounced and unexplained phenomena (i.e. anomalies) of choice, and subsequently the aggregate reaction of the stock market, that cannot be explained through normative theories and a more descriptive approach seems imperative.

Besides employing all fundamental elements of CPT and Markowitz theories, we also present in the appendix algorithmic approaches of Linear Programming (LP) and Mixed Integer Programming (MIP) optimization problems to perform the tests. These mathematical formulations introduce benefits regarding algorithmic performance, reduction of computational costs and subsequently computation time. The computational burden and complexity has been confronted by various coding techniques, such as introducing Irreducible Inconsistent Subsystems (IIS). An IIS is a subset of the constraints and variable bounds which reduce the initial infeasible system by removing a single (or multiple) constraint(s) or bound(s), and thus the subsystem becomes feasible. This reduction does not affect the problems' optimal solutions because the algorithm searches and finds the least possible variables and bounds to be removed. Further research on this topic could be performed within the boundaries of machine learning and more specifically upon variable importance, but this is something beyond the scope of this paper.

In the empirical application we conduct both in-sample and out-of-sample tests to assess the diversification benefits of alternative asset classes. Using individual commodity futures, FX rates, a U.S. real estate index and four precious metals, we show that depending on their attitudes towards risk, U.S. investors can improve the risk-adjusted performance when augmenting their traditional portfolios. In particularly, we found that commodities improve the investment opportunity set of CPT, Markowitz and LA type investors. Real estate improves the optimal portfolios of Markowitz investors, while CPT, RN and LA investors benefit from the inclusion of precious metals in their portfolios.

The first contribution of this work to the relevant literature is that we choose to apply primarily CPT and Markowitz, as well as additional types of preferences and attitudes towards risk on the cross section of our dataset. These six different personae have been the most pronounced and this is the reason why we employ them all. Based upon their application, we construct the so-called "behavioral", new and auxiliary datasets for each persona. These datasets can be elaborated in order to elicit preferences (and heuristics) and eventually the way these investors diversify their portfolios. We do so both for the Traditional and the Augmented portfolios. One key point is that we apply all the fundamental elements of the Cumulative Prospect Theory context on the crosssection, while so far the literature has payed little attention on that, especially when it comes to the empirical application of the objective probabilities' subjective distortion. Another contribution is that we sed light on the question whether alternative asset classes should be included in investors' portfolios. Until now, the results in the literature are mixed. Moreover, these four alternative asset classes (Commodities, FX, Precious Metals and Real Estate) have not been extensively studied in the Behavioral Finance literature, let alone when it comes in combination with specific attitudes towards risk. Finally, another contribution is the development of novel Mixed Integer and Linear Programs that help us solve the optimization problems, both in-and out-of-sample, tailored made for each investor type while they introduce significant algorithmic efficiency.

We assume that our data are i.i.d. and not normally distributed, something that can easily be verified from our dataset through descriptive statistics (Tables 6 and 7). We construct optimal portfolios non-parametrically to account for the positive skewness and kurtosis of returns, and compare both in-sample as well as out-of-sample, the performance of the traditional portfolio (T) with the enhanced (E) portfolio, for all investor personae, in all four experiments.

2. Methodology

2.1 Cumulative Prospect Theory type of preferences

The first version of Prospect Theory (henceforth PT) is described in the seminal paper of Kahneman and Tverksy (KT, 1979). It presents a descriptive theory of decision-making on prospects (i.e. probability distributions over monetary outcomes) under uncertainty, in a laboratory setting. More specifically, it is a positive economic model that asserts how investors *do behave* in contrast with normative economic models that assert how investors *should behave.* However, despite all its essential insights, the model presented in the paper can be applied only on prospects with two possible outcomes and also it does not always satisfy First Order Stochastic Dominance (FSD) (i.e. an individual can choose a dominated prospect).

The aforementioned drawback was tackled by an advancement in the original theory, published some years later, again from Tversky and Kahneman (TK, 1992), under the name Cumulative Prospect Theory (CPT). Now, gambles (or prospects) can have potentially infinite number of outcomes and Stochastic Dominance (SD) is satisfied. CPT is usually applied in economic and/or financial analysis.

To see how CPT works consider the following prospect

$$(x_{-m}, p_{-m}; \dots; x_{-1}; p_{-1}, x_o, x_1; p_1, \dots, x_n; p_n)$$

which should be read as "gain" x_{-m} with probability of occurrence p_{-m}. Moreover, $x_i < x_j \; for \; i < j$, and usually $x_o = 0$ (the reference point). Finally, $\sum_{-m}^{n} p_i = 1$ and in all our experiments we specifically employ the empirical probability; $p = 1/M$, where M is the number of assets included in the portfolio under elaboration.

The difference between CPT and the usual Expected Utility (EU) framework is found on four points. The first two are that under usual EU, an individual evaluates a prospect based on her utility function U (usually globally concave and everywhere differentiable) and by also taking into consideration the true probability distribution of the monetary outcomes. Let W be the current wealth level of the individual, thus she evaluates a prospect by computing

$$\sum_{i=-m}^{n} p_i U(W + x_i)$$

given that she is interested in her final wealth position.

On the other hand, a CPT agent distorts the objective probability distribution in a subjective way and she also employs an S-shaped value function (convex over the losses domain and concave over the gains, where the graph is steeper near the origin from the losses side and thus not differentiable at the origin). This is done initially by employing probability weighting functions (pwf) w+, w^{-} (or simply decision weights) for the gains and the losses domain respectively. Thus, an CPT agent evaluates a gamble through

$$\sum_{i=-m}^{n} \pi_i V(x_i)$$

where π refers to the so-called "capacities"

$$\pi_n^+ = w^+(p_n) \quad and \quad \pi_{-m}^- = w^-(p_{-m})$$

$$\pi_i^+ = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \quad where \quad 0 \leq i \leq \ n-1$$

and

$$\pi_i^- = w^-(p_{-m} + \dots + p_i) - w^-(p_{-m} + \dots + p_{i-1}) \quad where \quad 1-m \leq i \leq \ 0$$

Tverksy and Kahneman (1992) propose the following two functional forms

$$w^+(p) = \frac{p^{0.61}}{[p^{0.61} + (1-p)^{0.61}]^{\frac{1}{0.61}}} \quad and \quad w^-(p) = \frac{p^{0.69}}{[p^{0.69} + (1-p)^{0.69}]^{\frac{1}{0.69}}}$$

and

$$V^P(x) = \left\{ \begin{array}{cc} \mathrm{x}^a \quad if \quad x \geq 0 \\ -\lambda(-x)^a \quad if \quad x < 0 \end{array} \right\}$$

where a < 1 and λ > 1 is the loss aversion coefficient (λ = 2.25, because losses 'hurt' twice more than gains, and *a* = 0.88, TK 1992). The loss aversion coefficient is also "responsible" for the "intensity" of the kink at the origin. The higher its value the steeper the graph and hence the greater the agent's sensitivity (aversion) to losses. The *a*, *λ* values where estimated through actual decision-making lab experiments (TK, 1992) and refer to the median subject. Note that *v(0)* = w+(0) = w^-(0) = 0 *and* w+(1) = 1 = w^-(1). Given that agents distort objective probabilities, they usually result in over-weighting small probabilities of large gains and losses and in under-weighting moderate and high probabilities of small and intermediate gains and losses. This the reason why they result into inverse S-shaped functional forms for the pwf. In other words, the outcomes in the tails of the distribution, namely the extreme ones, are being over-weighted. The phenomenon of objective probability distortion together with different risk attitudes in the losses and in the gains domain, are compounding and not offsetting (Baucells and Heukamp, 2006). Finally, it is important to note that when applying the probability weighting, FSD is not violated. A large volume in the relevant literature associates this probability distortion with cognitive biases, as well as with numerous documented phenomena of choice that are not provided nor supported by the standard economic theory.

The third difference between CPT and EU is that under CPT, the agent derives utility from difference in gains and losses and not final wealth positions. Thus, we can say that the carriers of value are gains and losses. The fourth and final difference is that the curvature of the CPT's value function implies a dual meaning. Concavity implies risk aversion over gains, while convexity risk seeking over losses. A risk averse (avoiding unfair gambles) agent would always accept a certain payment even lower to the expected payoff of a gamble, rather than taking the gamble itself. Oppositely, a risk seeking (taking unfair gambles) agent would always accept to take a bet even if the guaranteed payment was higher of the certainty equivalent. CPT argues that decision makers are keen to exhibit both behavioral attitudes depending on the situation they find themselves. Hence, CPT provides the "context" for agile and not intransigent decision makers.

2.2 Markowitz type of preferences

Markowitz (1952) suggested that in the neighborhood of their current wealth level, investors employ a reverse S-shaped utility function. He concluded to this type of utility function after the observation that it is very common, among individuals who gamble, to buy both lottery tickets and insurance. Hence, the utilities they employ could not belong to the family of strictly (or weakly) concave utility functions but they should also exhibit convex segments. Thus, as a succession to the work of Friedman and Savage (1948) he proposed a functional form of a reverse S-shape utility (value) function.

$$V^M(x) = \left\{ \begin{array}{ccc} \mathrm{x}^a & if & x \geq 0 \\ -\lambda(-x)^a & if & x < 0 \end{array} \right\}$$

where $a > 1$

Markowitz's functional form can be easily derived from the functional form proposed by TK (1992), if the exponent a is strictly above 1. Moreover, the inclusion of the coefficient of loss aversion does not affect neither the general shape of the utility nor our subsequent analysis. Nevertheless, we select $\lambda = 1$ (neutrality towards aversion to losses) in order to remain consistent with Markowitz's initial theory.

2.3 Loss Averse (Myopic) type of preferences

Bernatzi and Thaler (henceforth BT, 1995) and Barberis, Huang and Santos (henceforth BHS, 2001) suggest that most investors' initial concern is to avoid losses (temporal, cross-sectional narrow framing), anyhow. They argue that this phenomenon stems from temporal and cross-sectional narrow framing (examining something in isolation). Moreover, they argue that this is the dominant behavior of the market's agents and many of the observed phenomena of choice (exuberant trading, equity premium puzzle, "house money" effect etc.) can be examined under the loss aversion prism. When investors succeed in avoiding losses, subsequently their interest shifts on expected returns and future cash flows. Thus, the value function a loss averse investors employs, can take a simple form, which is the following:

$$V^{LA}(x) = \left\{ \begin{array}{ccc} \mathrm{x} & \mathrm{if} & \mathrm{x} \geq 0 \\ -\lambda(-x) & if & x < 0 \end{array} \right\}$$

Again, the "penalty" for losses is λ, which is the same loss aversion coefficient as in CPT (experimental studies of attitudes to timeless gambles). BT estimate it around 2.5, a little higher than the initial estimate of TK (1992). Finally, one can see this type of investor persona, employs a simple decision shortcut (or heuristic). It could be sensible to see it this way because heuristics make the decision making process hugely redundant in complexity, and thus by declaring aversion to losses things are made a lot easier when forming optimal portfolios,while more sophisticated schemes are turned off. Nevertheless, it is a fact that the purpose of optimizing any decision-making process is to prone it to be as simple as possible and hence avoiding decision-making paralysis (or equivalently stemming from over-choice) (Schwartz, 2004). The term "myopic" refers to the work of BT, where such type of investors combine loss aversion with frequent valuations of their portfolios (daily, weekly or monthly), as the ones in our work where valuations are made on a monthly basis.

3. Empirical Application

3.1 Statistical Tests for Markowitz and CPT Stochastic Spanning

By taking into consideration all the above, and by following Arvanitis et.al. (2017, 2018), we form the test statistic of StSp. We employ the empirical distribution function F_T, which is a consistent estimator of F (the cdf of lb) and it is associated with the random element $(Y_t)_{t=1,\dots,T}$. In empirical applications the cdf F and the temporal dependence of the underlying process are latent. An initial problem with cdf F is that it can be unknown and a second problem is that the optimizations may be infeasible. Hence, its empirical estimate tackles these two interconnected problems. The following random variable, which is the scaled empirical analogue of p(F), plays the role of the test statistic.

The null and the alternative hypotheses of the asymptotically exact, feasible and consistent statistical test take the following forms:

$$p_T := \sqrt{T}p(F_T) = \sqrt{T}max_{i=1,2,3,4}sup_{\lambda\in\Lambda}sup_{z\in A_1,A_2}inf_{\kappa\in K}P_i(z,\lambda,\kappa,F_T) \tag{1}$$

The null and the alternative hypotheses of the asymptotically exact, feasible and consistent statistical test take the following forms:

$$H_0 : p_T = 0 \quad \& \quad H_A : p_T > 0 \tag{2}$$

The empirical joint cumulative distribution function which is constructed from our sample and

is the unconstrained maximum likelihood (ML) estimate of F is :

$$F_T = T^{-1}\sum_{t=1}^{T} 1(x_t \leq x) \quad where \quad 1(x_t \leq x) = \begin{cases} 1 & \text{if} \quad x_t \leq x \\ 0 & \text{if} \quad x_t > x \end{cases}$$

We assume that the returns' distribution is a latent stochastic process, with a continuous cdf $F : R^N \rightarrow [0,1]$ and with a finite covariance matrix of full rank (N). If the null hypothesis holds true, spanning occurs. On the contrary, if the alternative hypothesis holds true, spannning does not occur. In general, we can say that the null hypothesis holds if the enlargement (or reduction) of the choice set does not change the efficient set, which is a subset (not necessarily a proper one) of the choice set. Moreover, it would be useful to reformulate the test statistic in terms of expected utility. We do so in order to include the two different forms of the value functions conjugated with the pwf. Thus, we have:

$$p_T := \sqrt{T} sup_{\lambda\in\Lambda; v\in V^M, V^P; w\in W^-, W^+} inf_{\kappa\in K} E_{F_T}\left[v(X^T\lambda) - v(Y^T\kappa)\right] \quad (3)$$

In equation (17), X represents the Enhanced portfolio, while Y represents the Traditional. Hence, we are interested in testing whether *Y* spans *X*.

In order to combine StSp with the related family of utilities and pwf, we have the following notions. StSp occurs, in the case of S-shaped value function together with the pwf, if no satiable, risk averse for gains and risk seeker for losses type of investor benefits from the enlargement of the traditional opportunity set. On the other hand (reverse S-shaped value function and pwf), we have that StSp occurs if no satiable, risk averse for losses and risk seeker for gains type of investor benefits from the enlargement of the traditional opportunity set.

3.2 Description of data

We apply our models to test whether commodities, FX, real estate or precious metals should be included in portfolios consisting of stocks, bonds and cash in order to improve aggregate performance. More specifically, we use as benchmark assets ("traditional" assets) the S&P 500 Total Return Index, the Barclays US Aggregate Bond Index, and the 3-Month T-Bill as the risk-free rate (cash), the US Corporate AAA and BAA middle rate, ten different S&P indices (for example energy, health, technology, materials, utilities etc.) We use data on monthly closing prices obtained from Datastream and Bloomberg. The dataset spans the period 31/1/1994-30/4/2020, for a total of 316 monthly returns.

Tables 1, 2 & 3 report summary statistics for the performance of the benchmark assets, as well as for the commodities, the foreign exchange, the real estate and the precious metals.

Table 1: Descriptive Statistics (whole sample)

Entries report the descriptive statistics on monthly returns for the alternative asset classes that proxies the benchmark ("traditional") asset universe and the enhanced with commodities, foreign exchange (FX), real estate and precious metals. Data spans the period 31/1/1994 - 30/4/2020

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
US TREASURY CONST MAT 30 YEAR (D) - MIDDLE RATE	0,0037	0,0012	0,1382	-0,8069
US TREASURY CONST MAT 5 YEAR (D) - MIDDLE RATE	0,0028	0,0016	0,2957	-1,1545
US CORP BONDS MOODYS SEASONED AAA - MIDDLE RATE	0,0044	0,0012	0,1141	-1,0839
US CORP BONDS MOODYS SEASONED BAA - MIDDLE RATE	0,0052	0,0011	-0,0299	-1,1719
Bloomberg Barclays U.S. Aggregate USD - Average price	0,0001	0,0104	-0,2643	1,0371
S&P500 ES HEALTH CARE - TOT RETURN IND	0,0105	0,0431	-0,3378	0,3660
S&P500 ES CONSUMER DISCRETIONARY - TOT RETURN IND	0,0093	0,0516	-0,1809	1,6516
S&P500 ES CONSUMER STAPLES - TOT RETURN IND	0,0086	0,0362	-0,4163	1,4993
S&P500 ES INDUSTRIALS - TOT RETURN IND	0,0083	0,0513	-0,5748	1,8640
S&P500 ES INFO TECHNOLOGY - TOT RETURN IND	0,0127	0,0719	-0,3438	1,2388
S&P500 ES MATERIALS - TOT RETURN IND	0,0074	0,0586	-0,0399	1,5167
S&P500 ES COMM. SVS - TOT RETURN IND	0,0061	0,0559	0,2878	3,2960
S&P500 ES UTILITIES - TOT RETURN IND	0,0072	0,0439	-0,5118	0,7757
S&P500 ES FINANCIALS - TOT RETURN IND	0,0078	0,0622	-0,6713	3,0048
S&P500 ES ENERGY - TOT RETURN IND	0,0074	0,0618	-0,3933	4,6310
ICE-BRENT CRUDE OIL TRc1 - SETT. PRICE	0,0068	0,0945	-0,6220	4,4480
CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0018	0,0509	-0,4903	1,9342
CME-FEEDER CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0021	0,0444	-0,3445	1,5193
CME-LEAN HOGS COMP. CONTINUOUS - SETT. PRICE	0,0071	0,1118	0,2235	1,1028
CBT-CORN COMP. CONTINUOUS - SETT. PRICE	0,0035	0,0821	-0,1645	0,9930
CBT-SOYBEANS COMP. CONT. - SETT. PRICE	0,0033	0,0731	-0,4592	1,4217
CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE	0,0049	0,0887	0,5980	1,6038
Crude Oil WTI NYMEX Close M23 U\$/BBL	0,0056	0,0902	-0,4835	2,6954
CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE	0,0056	0,0451	0,1733	1,2381
CMX-SILVER 5000 OZ CONTINUOUS - SETT. PRICE	0,0067	0,0818	0,0906	0,8345
CSCE-COTTON #2 CONTINUOUS - SETT. PRICE	0,0034	0,0874	-0,1947	1,2866
CSCE-COFFEE ’C’ CONTINUOUS - SETT. PRICE	0,0067	0,1079	1,0447	2,7274

Table 2: Descriptive Statistics (whole sample) continued

Entries report the descriptive statistics on monthly returns for the alternative asset classes that proxies the benchmark ("traditional") asset universe and the enhanced with commodities, foreign exchange (FX), real estate and precious metals. Data spans the period 31/1/1994 - 30/4/2020

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
CSCE-COCOA CONTINUOUS - SETT. PRICE	0,0063	0,0888	0,3563	0,9885
CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE	0,0042	0,0940	0,3341	0,7742
S&P UNITED STATES REIT U\$ - TOT RETURN IND	0,0004	0,0239	0,7920	5,0453
CANADIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0239	0,7920	5,0453
DANISH KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0273	0,3156	1,1177
JAPANESE YEN TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0302	-0,2430	2,4272
NORWEGIAN KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0014	0,0309	0,4380	1,2649
SOUTH AFRICA RAND TO US \$ NOON NY - EXCHANGE RATE	0,0064	0,0450	0,6399	1,3514
SWEDISH KRONA TO US NOON NY - EXCHANGE RATE	0,0010	0,0305	0,1646	0,5371
SWISS FRANC TO US \$ NOON NY - EXCHANGE RATE	-0,0009	0,0291	-0,0003	1,5719
AUSTRALIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0007	0,0338	0,8349	3,8874
NEW ZEALAND \$ TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0351	0,5707	1,9608
UK £ TO US \$ NOON NY - EXCHANGE RATE	0,0008	0,0243	0,5336	1,7642
INDIAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0030	0,0199	0,7235	3,5632
SRI LANKAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0044	0,0136	1,4274	8,5387
CHINESE YUAN TO US \$ NOON NY - EXCHANGE RATE	0,0009	0,0290	16,2481	279,9218
HONG KONG \$ TO US \$ NOON NY - EXCHANGE RATE	0,0000	0,0013	-0,8130	6,3969
SINGAPORE \$ TO US \$ NOON NY - EXCHANGE RATE	-0,0003	0,0164	0,5122	2,8569
THAI BAHT TO US \$ NOON NY - EXCHANGE RATE	0,0012	0,0310	2,4579	28,4361
SOUTH KOREAN WON TO US \$ NOON NY - EXCHANGE RATE	0,0021	0,0423	4,0250	41,7343
TAIWAN NEW \$ TO US \$ NOON NY - EXCHANGE RATE	0,0005	0,0155	0,3397	4,6853
MEXICAN PESO TO US \$ NOON NY - EXCHANGE RATE	3,3330	55,7453	17,6651	313,2405
Palladium U\$/Troy Ounce	0,0139	0,1015	0,4711	3,1720
London Platinum Free Market \$/Troy oz	0,0041	0,0617	-0,5432	3,1083
Gold, Handy & Harman Base \$/Troy Oz	0,0057	0,0449	0,1398	1,2947
Silver, Handy&Harman (NY) U\$/Troy OZ	0,0067	0,0804	0,1090	0,9638

Table 3: Descriptive Statistics

Entries report the descriptive statistics on monthly returns for the alternative asset classes. Data spans the period 1/1/2000 - 4/30/2020.

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
US TREASURY CONST MAT 30 YEAR (D) - MIDDLE RATE	0,0037	0,0012	0,1382	-0,8069
US TREASURY CONST MAT 5 YEAR (D) - MIDDLE RATE	0,0028	0,0016	0,2957	-11,545
US CORP BONDS MOODYS SEASONED AAA - MIDDLE RATE	0,0044	0,0012	0,1141	-10,839
US CORP BONDS MOODYS SEASONED BAA - MIDDLE RATE	0,0052	0,0011	-0,0299	-11,719
Bloomberg Barclays U.S. Aggregate USD - Average price	0,0001	0,0104	-0,2643	10,371
S&P500 ES HEALTH CARE - TOT RETURN IND	0,0105	0,0431	-0,3378	0,3660
S&P500 ES CONSUMER DISCRETIONARY - TOT RETURN IND	0,0093	0,0516	-0,1809	16,516
S&P500 ES CONSUMER STAPLES - TOT RETURN IND	0,0086	0,0362	-0,4163	14,993
S&P500 ES INDUSTRIALS - TOT RETURN IND	0,0083	0,0513	-0,5748	18,640
S&P500 ES INFO TECHNOLOGY - TOT RETURN IND	0,0127	0,0719	-0,3438	12,388
S&P500 ES MATERIALS - TOT RETURN IND	0,0074	0,0586	-0,0399	15,167
S&P500 ES COMM. SVS - TOT RETURN IND	0,0061	0,0559	0,2878	32,960
S&P500 ES UTILITIES - TOT RETURN IND	0,0072	0,0439	-0,5118	0,7757
S&P500 ES FINANCIALS - TOT RETURN IND	0,0078	0,0622	-0,6713	30,048
S&P500 ES ENERGY - TOT RETURN IND	0,0074	0,0618	-0,3933	46,310
ICE-BRENT CRUDE OIL TRc1 - SETT. PRICE	0,0068	0,0945	-0,6220	44,480
CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0018	0,0509	-0,4903	19,342
CME-FEEDER CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0021	0,0444	-0,3445	15,193
CME-LEAN HOGS COMP. CONTINUOUS - SETT. PRICE	0,0071	0,1118	0,2235	11,028
CBT-CORN COMP. CONTINUOUS - SETT. PRICE	0,0035	0,0821	-0,1645	0,9930
CBT-SOYBEANS COMP. CONT. - SETT. PRICE	0,0033	0,0731	-0,4592	14,217
CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE	0,0049	0,0887	0,5980	16,038
Crude Oil WTI NYMEX Close M23 U\$/BBL	0,0056	0,0902	-0,4835	26,954
CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE	0,0056	0,0451	0,1733	12,381
CMX-SILVER 5000 OZ CONTINUOUS - SETT. PRICE	0,0067	0,0818	0,0906	0,8345
CSCE-COTTON #2 CONTINUOUS - SETT. PRICE	0,0034	0,0874	-0,1947	12,866
CSCE-COFFEE 'C' CONTINUOUS - SETT. PRICE	0,0067	0,1079	10,447	27,274
CSCE-COCOA CONTINUOUS - SETT. PRICE	0,0063	0,0888	0,3563	0,9885
CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE	0,0042	0,0940	0,3341	0,7742
S&P UNITED STATES REIT U\$ - TOT RETURN IND	0,0004	0,0239	0,7920	50,453
CANADIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0239	0,7920	50,453
DANISH KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0273	0,3156	11,177
JAPANESE YEN TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0302	-0,2430	24,272
NORWEGIAN KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0014	0,0309	0,4380	12,649
SOUTH AFRICA RAND TO US \$ NOON NY - EXCHANGE RATE	0,0064	0,0450	0,6399	13,514
SWEDISH KRONA TO US NOON NY - EXCHANGE RATE	0,0010	0,0305	0,1646	0,5371
SWISS FRANC TO US \$ NOON NY - EXCHANGE RATE	-0,0009	0,0291	-0,0003	15,719
AUSTRALIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0007	0,0338	0,8349	38,874
NEW ZEALAND \$ TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0351	0,5707	19,608
UK £ TO US \$ NOON NY - EXCHANGE RATE	0,0008	0,0243	0,5336	17,642
INDIAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0030	0,0199	0,7235	35,632
SRI LANKAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0044	0,0136	14,274	85,387
CHINESE YUAN TO US \$ NOON NY - EXCHANGE RATE	0,0009	0,0290	162,481	2,799,218
HONG KONG \$ TO US \$ NOON NY - EXCHANGE RATE	0,0000	0,0013	-0,8130	63,969
SINGAPORE \$ TO US \$ NOON NY - EXCHANGE RATE	-0,0003	0,0164	0,5122	28,569
THAI BAHT TO US \$ NOON NY - EXCHANGE RATE	0,0012	0,0310	24,579	284,361
SOUTH KOREAN WON TO US \$ NOON NY - EXCHANGE RATE	0,0021	0,0423	40,250	417,343
TAIWAN NEW \$ TO US \$ NOON NY - EXCHANGE RATE	0,0005	0,0155	0,3397	46,853
MEXICAN PESO TO US \$ NOON NY - EXCHANGE RATE	33,330	557,453	176,651	3,132,405
Palladium U\$/Troy Ounce	0,0139	0,1015	0,4711	31,720
London Platinum Free Market \$/Troy oz	0,0041	0,0617	-0,5432	31,083
Gold, Handy & Harman Base \$/Troy Oz	0,0057	0,0449	0,1398	12,947
Silver, Handy&Harman (NY) U\$/Troy OZ	0,0067	0,0804	0,1090	0,9638

3.3 Subsampling Procedure : In-sample Analysis

The following consistent subsampling procedure is formulated in order to provide the critical values for the rejection areas of the test statistic. We do so by employing block bootstrapping with overlapping blocks of data which include monthly raw returns. This is done because the real c.d.f F is unknown and thus the test statistic $p(F)$ cannot be estimated. We then resample the blocks and generate the bootstrap sample. The distribution of subsample test scores can be described by the following c.d.f. and quantile function, where $a \in [0,1]$ is the significance level, which we keep it fixed at 5%:

$$S_{T,b_T}(y) = (T - b_T + 1)^{-1} \sum_{t=1}^{T-b_T+1} 1(p_{b_T;T,t} \leq y) \tag{4}$$

and

$$q_{T,b_T}(1-a) = inf_y \left\{ y : S_{T,b_T}(y) \geq 1-a \right\} \tag{5}$$

In our in-sample analysis, we assume that the asymptotic decision rule is to reject the null H0: $p_T(F) = 0$ against the alternative $H1: p_T(F) > 0$ at significance level a, if and only if $p_T(F) > q(p_\infty, 1 - a)$, which is the $(1 - a)$ quantile of distribution p_∞ for any a. The null hypothesis is that the traditional set spans the enhanced one, while the alternative hypothesis is that there are some portfolios that are not spanned by the traditional assets. Due to the latency of the real c.d.f F, the aforementioned rule exhibits infeasibility thus we rely on an approximation based on a subsampling procedure.

More specifically, we choose the length b_T with $b_T = \lfloor T^c \rfloor, c \in [0.6, 0.9]$, with a step of 0.1 (Arvanitis et.al. , 2017), of the overlapping subsamples $(X_s)_{s=t}^{t+b_T-1}, t = 1, \ldots, T - b_T + 1$ and evaluate the test statistic on each one, thereby obtaining $p_{b_{T,T,t}}, t = 1, \ldots, T - b_T + 1$, and hence resulting to the evaluation $q_{T,b_T}(1 - a)$ which is the $(1 - a)$ quantile of the empirical distribution $p_{bT},T,_t$ across all subsamples.

Given the above, we use a modified version of the decision rule which is to reject the null if and only if $p_T > q_{T,b_T}(1 - a)$. This results into an asymptotically exact and consistent test as long as the significance level a is appropriately chosen (in our empirical application it suffices that $a < 0.25$) and the subsampling rate b_T diverges to infinity at a slower rate than T. Moreover, in order to correct for any biases resulting from the different subsample sizes, we follow Arvanitis et al. (2017) and estimate a regression of the estimated critical values and the relevant subsample lengths. The authors argue that this procedure is consistent with the relevant limit theory, and show through MC simulations, that this method is more efficient and powerful in small samples, exactly as the one we employ. Finally, we use the estimated regression line evaluated at T in order to obtain the bias corrected critical value, proper for finite samples of realistic time series and cross sectional dimensions, in order to apply it to our modified decision rule.

In the real estate experiment, we find that $p_T^{CPT} = 0.14416 < q_{T,b_T,CPT} = 0.17009$ and $p_M^{Markowitz} = 0.00019 > q_{T,b_T,Markowitz} = 0.00013, p_T{}^{RiskAverse} = 0.12342 < q_{T,b_T,RiskAverse} = 0.14562, p_T^{Loss\ Averse} = 0.05887 > q_{T,b_T,LossAverse} = 0.03141$ and $p_T^{RiskNeutral} = 0.0000001 > q_{(T,b_T,RiskNeutral)} = -0.001887$. Thus, we reject the null hypothesis that the traditional asset classes span the augmented asset classes set, in favor of the alternative, for Markowitz, LA and RN type of investors. Thus the results of this in-sample-analysis indicate that the performance of traditional portfolios, consisting of bonds and indices, can be improved by including the S&P Real Estate index for some specific risk attitudes. Thus, some investors with Markowitz, LA and RN type of preferences could benefit from the augmentation.

In the commodities experiment, we find that $p^{CPT} = 0.339109 > q_{T,b_T,CPT} = 0.24743$ and $p_T^{Markowitz} = 0.00018 > q_{T,b_T,Markowitz} = 0.00003, p_T^{RiskAverse} = 0.00042 > q_{T,b_T,RiskAverse} = 0.00026, p_T^{LossAverse} = 0.09485 > q_{T,b_T,LossAverse} = 0.0001$ and $p_T^{RiskNeutral} = 0.0004 > q_{T,b_T,RiskNeutral} = — 0.0027$. Thus, we reject the null hypothesis that the traditional asset class spans the asset class, in favor of the alternative, for all investor types. Thus, some investors from the range of all these personas could benefit from the enhancement of their portfolios.

In the foreign exchange experiment, we find that $p_T^{CPT} = 0.45426 < q_{T,b_T,CPT} = 0.56474$ and $p_T^{Markowitz} = 0.00083 < q_{T,b_T,Markowitz} = 0.00095, p_T^{RiskAverse} = 0.00122 < q_{T,b_T,RiskAverse} = 0.00152, p_T^{LossAverse} = 0.04 > q_{T,b_T,LossAverse} = 0.001$ and $p_T^{RiskNeutral} = 0.0009 > q_{T,b_T,RiskNeutral} = — 0.01$. Thus, we cannot reject the null hypothesis only for CPT, Markowitz and RA investor type.

Hence, we can say that no investor of any of these three preferences' types could benefit from the addition of FX in their portfolios. However, this does not hold true for LA and RN types and some investors of these preferences' types could find the inclusion of FX beneficial.

Finally, in the precious metals experiment we find that $p_T^{CPT} = 0.65546 > q_{T,b_T,CPT} = 0.61399$ and $p_T^{Markowitz} = 0.00432 < q_{T,b_T,Markowitz} = 0.00566, p_T^{RiskAverse} = 0.00546 < q_{T,b_T,RiskAverse} = 0.00821, p_T^{LossAverse} = 0.025 > q_{T,b_T,LossAverse} = -0.1$ and $p_T^{RiskNeutral} = 0.5 > q_{T,b_T,RiskNeutral} = -0.2$. Thus, we reject the null hypothesis for the CPT, LA and RN type, hence some investors of these type could benefit from the addition of precious metals in their investing strategies.

3.4 Out-of-sample Analysis & Assessment

Backtesting, or out-of-sample testing, is the simulation of the performance of a strategy over an appropriate period of time and the analysis of the levels of profitability and risk. It is important to note that, the in-sample test results may differ from the out-of-sample ones. Thus, in all experiments, we are interested in the out-of-sample portfolios' performance of the six different investors' personae. We apply the relevant value functions and/or associated p.w.f. on the raw returns on each month and hence we obtain a "behavioral" new dataset. Essentially, each raw return has turn into a "behavioral" value that reflects the impression an investor has when s/he looks at the returns (or other types of data). These impressions affect significantly the way investors place the fractions of their wealth on assets (i.e. when they decide about the optimal weights in their traditional and enhanced portfolios). The rolling window analysis covers 120 months (10 years), starting from 1/1/2000, and thus we construct portfolios based on the behavioral information up to time t and we then reap their returns, on the actual dataset, at time t+1. We form optimal portfolios separately for the traditional and the enhanced The clock is advanced and the realized returns of the optimal portfolios are determined from the actual raw returns of the various assets. This procedure is repeated for all subsequent monthly returns in our dataset till 4/30/2020.

In all our experiments we followed the reasoning that investors keep track of the aggregate monthly market performance, when they are about to project future performance. They look at historical data and then mimic their trading strategy through back-testing. This procedure is well known to be applied by individual investors as well as investment funds. The important point is that given the monthly behavior of the overall market index (monthly closing level), individual investors, team of investors and/or mangers can regard all the previous period (120 months) as a gains or a losses period, respectively. This performance is each month either soaring or plummeting. Thus, when we have a negative monthly outcome, the former period (120 months) is regarded as losses and when the outcome is positive, as gains. This is crucial in order to be able to apply CPT, LA and Markowitz approaches in our experiments.

3.4.1. Case 1: Real Estate

In this first experiment it is interesting to notice that despite the differences in aggregate returns for all investor types except one, the Traditional (T) and the Enhanced (E) perform more or less the same, on average. One possible explanation may be that (T) contains 15 assets which are bonds and indices, while the introduction of the Real Estate index, in order to form (E), does not affect the aggregate performance significantly. Thus, with an average return of 0.04%, someone could argue that an investor, except for CPT and LA, would remain indifferent regarding the inclusion of the S&P U.S. REIT or not, and probably she would be better off by avoiding the specific asset's volatility and covariance with the other assets. Moreover, the portfolios' optimal diversification appears to avoid the market's turmoil of 2008 and no need for recovery was needed. For the CPT and LA types we have the interesting outcomes that CPT performs poorly, no matter she does or she does not include real estate in her portfolio, with (T) performing slightly better. Her behavioral pattern reacts exactly in the opposite way, compared to Markowitz, RA and RN, most likely because of the curvature of the value functions accompanied with the pwf. The winner of this experiment is the LA type where (T) outperforms significantly (E) throughout the whole period. It seems that the loss averse coefficient provides some short of "stability" in the aggregate performance of (E) and facilitates the over-performance of the (T).

3.4.2 Case 2: Commodities

In this experiment, various interesting outcomes emerge. For the CPT investor the (E) outperforms the (T) with the latter exhibiting losses throughout the whole period. For the Markowitz type, both portfolios perform more or less the same, while for the RA again the picture is vague with both (E) and (T) exhibiting about the same aggregate returns. She, the RA investor, is constantly looking for safety so she avoids positive skewness, she prefers assets with less kurtosis, as the traditionals compared to the commodities. Smaller kurtosis indicates less outliers thus safer assets. Now, (E) contains 29 assets in total, where 14 are commodities. Their average return is 0.4% and most of them are positively skewed, encouraging bets for the short term. Positively skewness is one of the key features a CPT agent is aiming at because it signals gains in the short term. On the other hand, the Markowitz type is willing to bet on negatively skewed assets because she is looking for larger returns in the long term. However, the small number of assets exhibiting negative skewness does not provide the necessary compound return in order to outmatch (T). Again, the RN investor demonstrates a similar neutrality in her aggregate returns, while the LA agent seems willing to include commodities in her portfolio. This time the naive investor has her portfolios both plummet, throughout the period under examination, however (T) is performing better compared to (E).

3.4.3 Case 3: FX

In this experiment, we can observe the Traditional portfolio outperform the Enhanced, for the

LA, RA and Markowitz investor while for CPT it also does better, for the largest part of the period. Now the (E) contains 33 assets. For M, RN and RA the two portfolios exhibit no significant differences, with the RN case being not clear because both portfolios produce more or less equivalent aggregate returns. Again, the LA investor favors more the traditional asset group and this option seems to pay her back. For all investor types, besides presenting an average return of 0.1%, the FX assets exhibit extreme kurtosis, on average. Thus, the tails of the leptokurtic distribution are thicker, revealing that there exist asset returns at the extremes or to state it more simply, the distribution produces a significant number of outliers. CPT, M and RA investors exhibit the so-called kurtosis aversion, with the CPT being more averse. Finally, the naive investor presents significant positive spread, in terms of aggregate performance, between (E) and (T) with (E) being greater.

3.4.4. Case 4: Precious Metals

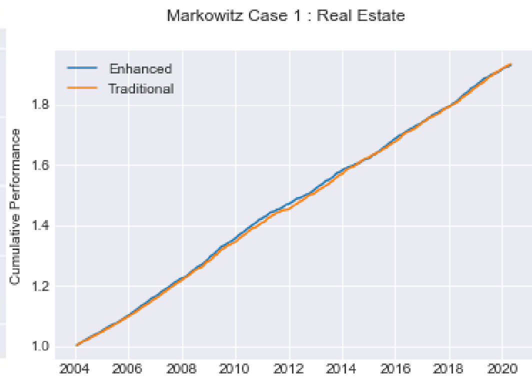
For the four precious metals, treated all in one group, we obtain analogous results. The CPT's Enhanced portfolio seems to over-perform the Traditional but again in the losses domain. Markowitz type is once again not interested in including new assets. Hence, she places little weight on them and this is the reason why both portfolios perform more or less the same. This is also true for the RA type, where she similarly exhibits reluctance for these assets and prefers more conservative choices. The average return of the four precious meals is about 0.7% , with variances on average 0.07. Now, kurtosis declares less accumulation of observations in the tails of the distributions, thus less outliers, thus fewer potentials for extreme gains and/or losses. We see that the RA, as well as the Markowitz type, seem both to avert higher kurtosis compared to the traditional assets and rely on more homogeneous assets. For the LA type, the situation is not so clear. She exhibits a steep peak in aggregate performance, however (T) seems to perform better on average because of high returns and no severe losses. The naive investor is facing significant losses when she bets also on precious metals, with her (T) performing better than (E), however both are losing.

3.5 Non-parametric tests

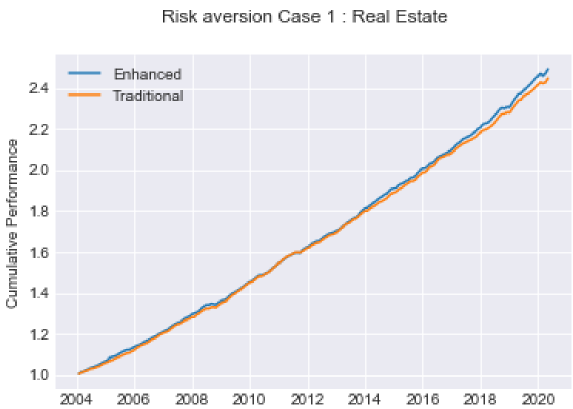
There is a significant number of pairwise SD tests presented in the literature as for example Linton et al. (2005), Barret and Donald (2003), Davidson and Duclos (2013) etc. For our non-parametric tests, we employ the Scaillet and Topaloglou (2010) SD efficiency (SDE) tests because they can be also implemented in a non-parametric setting even for StSp (Table 8). Their advantage is that they exhibit two important elements, which are well suited for our experiments. First, they allow for correlated samples and second, they also allow for time dependent data and do not assume i.i.d. returns. The p-values of the tests are presented in Table 8. The null hypothesis in all cases is that the Traditional SSD the Enhanced (or Augmented) portfolio. SSD is a perquisite in order for the results to be aligned with the results of second order StSp. Thus, the null can be rejected only in the Real Estate experiment for the Markowitz type. Moreover, it is also rejected for the CPT type in Commodities and Precious Metals. Interestingly, these results are not aligned with the results of the backtesting procedure. For the RA type, the traditional spans the enhanced in all cases, while for LA type we reject the null in all cases. Again, these results are not in covenant with the out-of-sample tests, something we know apriori it can occur. P-values are obtained through block bootstrapping with overlapping blocks. Block bootstrapping extends the non-parametric i.i.d. bootstrapping to time series data. Data are divided into blocks and these blocks are re-sampled in order to mimic the time dependent structure of the original data. Subsequently, we simulate the associated p-values.

Figure 1: Cumulative performance in the first case of real estate

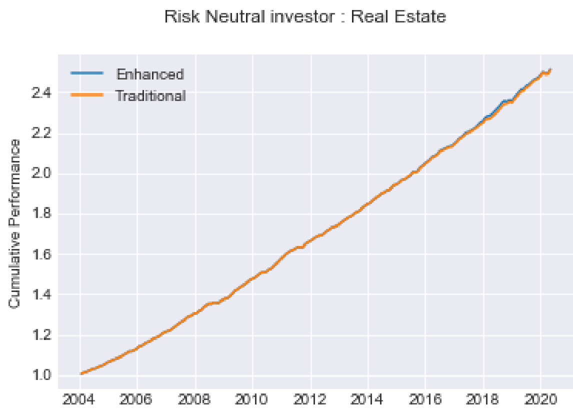
(a) CPT type of investor



(b) Markowitz type of investor

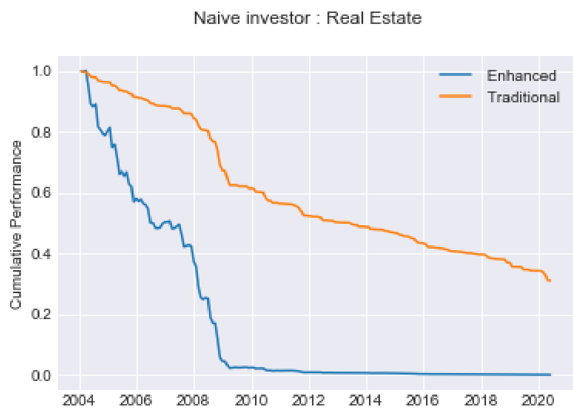


(c) Risk averse type of investor



(d) Risk neutral type of investor

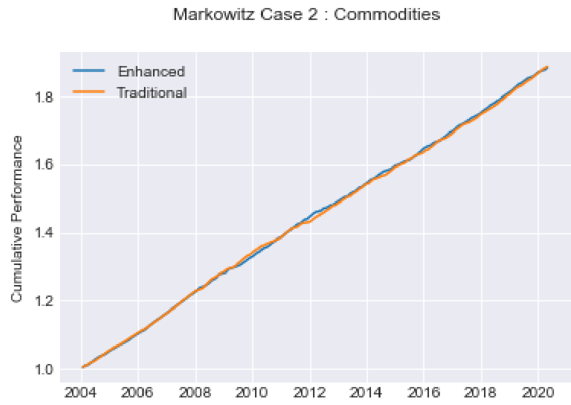
(e) Loss averse type of investor



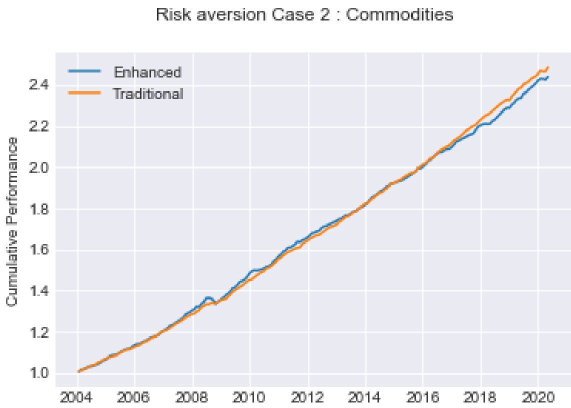
(f) Naive type of investor

Figure 2: Cumulative performance in the second case of commodities

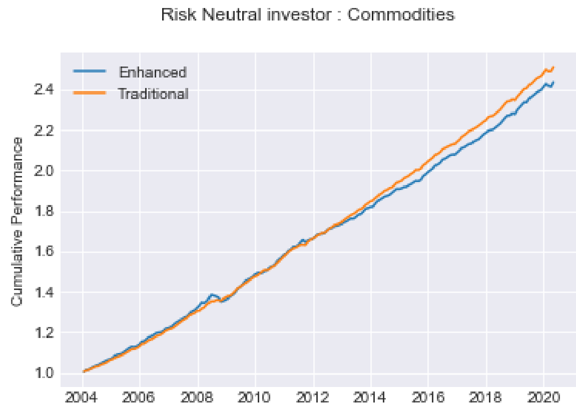
(a) CPT type of investor



(b) Markowitz type of investor



(c) Risk averse type of investor

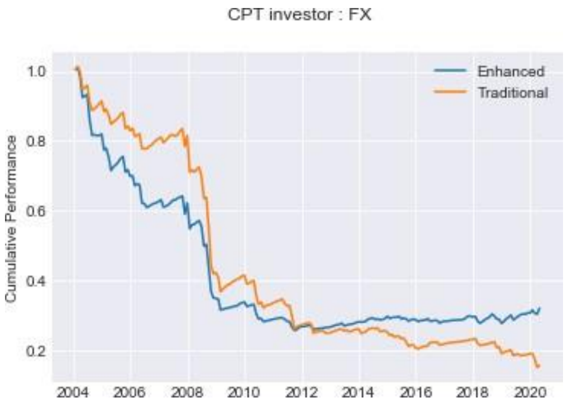


(d) Risk neutral type of investor

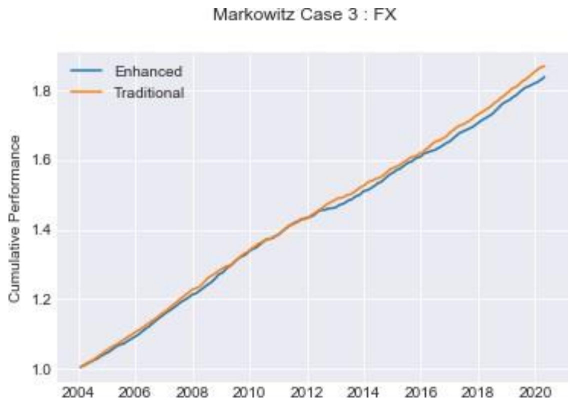
(e) Loss averse type of investor

(f) Naive type of investor

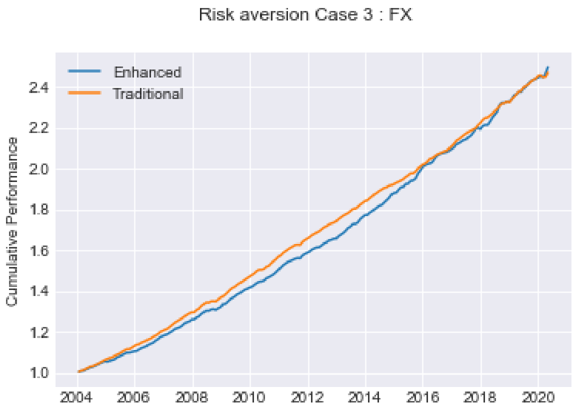
Figure 3: Cumulative performance in the third case of FX



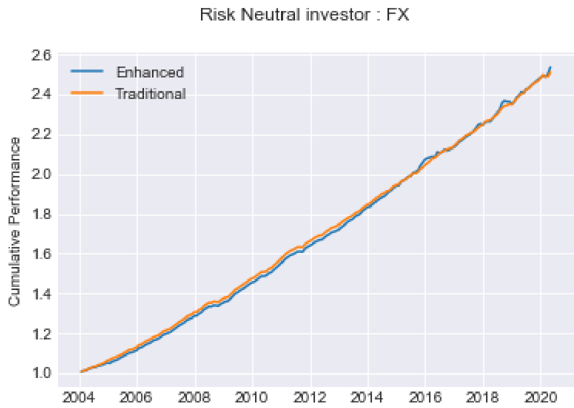
(a) CPT type of investor



(b) Markowitz type of investor

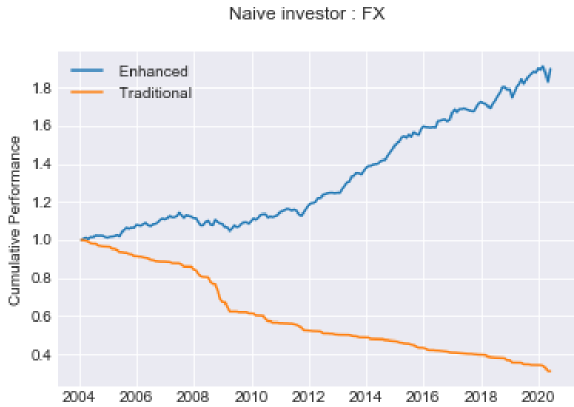


(c) Risk averse type of investor



(d) Risk neutral type of investor

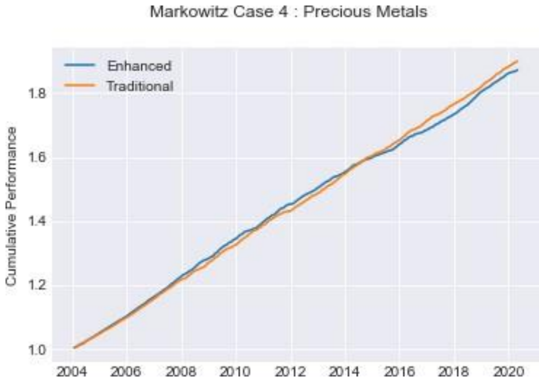
(e) Loss averse type of investor



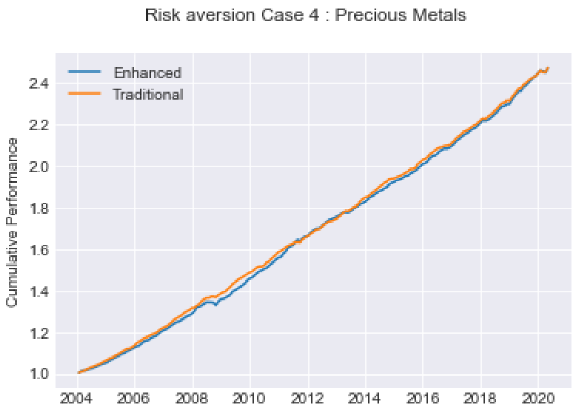
(f) Naive type of investor

Figure 4: Cumulative performance in the fourth case of precious metals

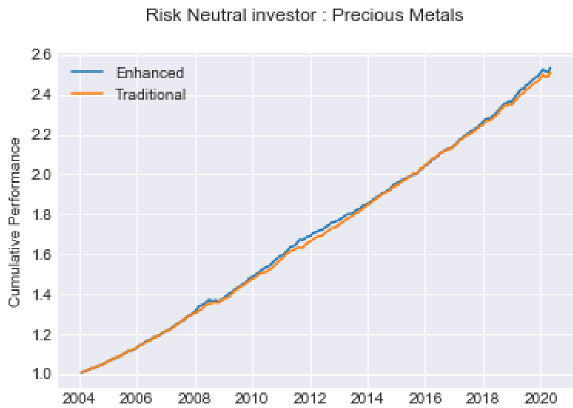
(a) CPT type of investor



(b) Markowitz type of investor



(c) Risk averse type of investor



(d) Risk neutral type of investor

(e) Loss averse type of investor

(f) Naive type of investor

Table 8: Scaillet & Topaloglou Non - parametric tests for SSD

The null hypothesis is that the Traditional portfolio SSD the Augmented, in every case. P-values are obtained through bootstrapping with overlapping blocks. We reject the null if the p-value is lower than 5%.

Significance level at 5%		p-value
Real Estate	CPT	0.0691
	Markowitz	0.0451
	Risk Averse	0.0537
	Loss Averse	0.0314
Commodities	CPT	0.0317
	Markowitz	0.0053
	Risk Averse	0.0859
	Loss Averse	0.0001
Foreign Exchange	CPT	0.0812
	Markowitz	0.0654
	Risk Averse	0.0728
	Loss Averse	0.0001
Precious Metals	CPT	0.021
	Markowitz	0.0526
	Risk Averse	0.0612
	Loss Averse	0.0261

3.6 Parametric tests

We compute a number of commonly used parametric performance measures: the Sharpe ratio, the downside Sharpe ratio (DS)(Ziemba, 2005), the upside potential (UP) and downside risk ratio (Sortino and van den Meer, 1991), the opportunity cost (Simaan, 2013), the portfolio turnover (P.T.) and a measure of the portfolio risk-adjusted returns net of transaction costs (RL). Due to the fact that the assets' returns exhibit asymmetric return distributions, the downside Sharpe and UP ratios are more appropriate measures than the typical Sharpe ratio.

For the DS ratio, we first need to calculate the downside risk (downside variance) which is given by the formula:

$$\sigma_{P\_}^2 = \frac{\sum_{t=1}^{T}(min(x_t, 0))^2}{T-1}$$

where, x_t are those returns of portfolio P at day t below 0, i.e. those days with losses. To get the total variance we use: 2ap_, thus the DS ratio is

$$S_P = \frac{\overline{R}_P - \overline{R}_f}{\sqrt{2\sigma_{P\_}^2}}$$

where, $\overline{\overline{R_P}}$is the average period return of portfolio P and $\widetilde{\widetilde{R_f}}$ is the average risk free rate.

The DS ratio, removes any effects of upward price movement on standard deviation in order to focus on the distribution of the returns that are below a predefined threshold/target that is set by an investor (or fund) as a minimum required return. Its difference with Sharpe ratio is that it replaces the risk-free rate with the required return. In our experiments we assume that this required return is the average risk-free return of the whole period under examination.

The UP ratio compares the upside potential to the shortfall risk over a benchmark and is computed as follows. Let R_t be the realized daily return of portfolio P for $t = 1 ,...,T$ of the backtesting period, where T is the number of experiments performed and let p_t be respectively the return of the benchmark (risk free rate), which in our case is the one month T-bill riskless asset for the same period. Then we have,

$$UP \quad ratio = \frac{\frac{1}{T_1}\sum_{t=1}^{T_1} max(R_t - p_t, 0)}{\sqrt{\frac{1}{T_2}\sum_{t=1}^{T_2}(max(p_t - R_t, 0))^2}} \quad , T = T_1 + T_2$$

The numerator of the above ratio is the average excess return over the benchmark and thus it reflects the upside potential. In the same sense, the denominator measures downside risk, i.e. shortfall risk over the benchmark.

Next, we compute the P.T. to get a feeling of the degree of rebalancing required to implement each one of the investment strategies under examination. For any portfolio strategy *P*, the portfolio turnover is defined as the average of the absolute change of weights over the *T* rebalancing points in time and across the M available assets, i.e.

$$P.T. = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{M}\left(\mid w_{Pi,t+1} - w_{Pi,t} \mid \right)$$

where $w_{P_i,t+1}$, $w_{P_i,t}$ are the optimal weights of asset *i* under strategy P (Traditional or Enhanced) at time t and t+1, respectively.

We also evaluate the performance of the portfolios under the risk-adjusted returns measure, which is net of transaction costs, proposed by DeMiguel et al. (2009). It indicates the way that the proportional transaction cost, generated by the P.T., affects the portfolio returns. Let *trc* be the proportional transaction cost, and $R_{P,t+1}$ the realized return of portfolio P at time t+1. The change in the net of transaction cost wealth NW_P of portfolio P through time is,

$$NW_{P,t+1} = NW_{P,t+1}(1 + R_{P,t+1})\left(1 - trc \times \sum_{i=1}^{M}\left(\mid w_{P,i,t+1} - w_{P,i,t} \mid \right)\right)$$

The portfolio return, net of transaction cost, is defined as,

$$RTC_{P,t+1} = \frac{NW_{P,t+1}}{NW_{P,t}} - 1$$

Let p_{Tr}, μ_{Aug} be the out-of-sample mean of monthly *RTC* with the traditional and opportunity set, respectively, and σ_{Tr}, σ_{Aug} be the corresponding standard deviations. Then, the return-loss measure is,

$$R_{Loss} = \frac{\mu_{Aug}}{\sigma_{Aug}} \times \sigma_{Tr} - \mu_{Tr}$$

It evaluates the additional return needed so that the Traditional performs equally well with the Enhanced. We follow the literature and use 35 basis points (bps), i.e. 0.35% , for the proportional transaction cost of stocks and bonds.

Finally, we use the concept of opportunity cost presented in Simaan (1993) to analyze the economic significance of the performance difference of the two optimal portfolios, in both experiments and for both investor types. Let R^i_{Tr} and R^i_{Aug} be the realized returns of the optimal Traditional and portfolio for every investor i. Then, the opportunity cost θ is defined as the return that needs to be added to (or subtracted from) RT_r, so that the investor is indifferent (in utility terms) between the strategies imposed by the two different investment opportunity classes

$$E\left[U\left(1 + R^i_{Tr} + \theta\right)\right] = E\left[U\left(1 + R^i_{Aug}\right)\right] \quad ; i = CPT, M, RA, RN, LA$$

A positive opportunity cost implies that an investor is better off if she includes additional assets in her portfolio, while a negative one implies that she would be worse off with the aforementioned inclusion. It is important to mention that the opportunity cost takes into account the entire probability distribution of portfolio returns and hence it is suitable to evaluate strategies even when the distribution is not normal. For the calculation of the opportunity cost we follow the literature and use the relevant *S-shaped*, *inverse S-shaped* and *globally concave* utility functions, consistent with SSD and associated with the investor personas.

Tables 9, 10, 11, and 12 report the parametric performance measures for the Traditional and the Enhanced portfolios, for all investor types. In these four experiments and although they are parametric they enrich the evidence obtained from the non-parametric SD measures. The higher the value of each one of these measures, the greater the investment opportunity for including the additional aforementioned assets, of real estate, commodities, FX and precious metals. For these experiments we follow the literature and use 35 bps of the transaction costs of stocks and bonds. We pay our attention on CPT, Markowitz and LA investors due to our interest through the Behavioral Finance perspective.

From the results, we can see that the inclusion of the real estate index into the opportunity set reduces both the Sharpe ratios and the downside Sharpe ratios for the CPT type, LA type and the RA type, while for the Markowitz type we observe a significant increase in both of these measures. This results in a decrease in the risk-adjusted performance (i.e. an increase in expected return per unit of risk) and hence contracts the investment opportunities of some RA, LA and CPT investors. On the contrary, for the Markowitz investor the investment opportunities are expanded. For commodities the picture is vague. We can see that the Sharpe ratios for CPT and Markowitz increase, while for the RA decrease. For the downside Sharpe ratio, increment is only present for the Markowitz type. Thus, we can say with certainty that for Markowitz type investment opportunities become wider. For the FX experiment and for all types both Sharpe, as well as downside Sharpe ration decrease, hence investment opportunities seem to fade away. Finally, for precious metals CPT investor benefits from investing on precious metals while Markowitz is being damaged and RA remains ambiguous. Regarding the UP ratio, we see that in the first experiment the inclusion of the real estate index only benefits the Markowitz investor. In the commodities experiment only the RA type results not being benefited. In the third (FX) experiment, we see that all investor types result into smaller upside potentials, while in the last experiment precious metals benefit only the CPT type. Furthermore, we can observe that portfolios with only traditional assets induce less portfolio turnover compared to the ones including the real estate index for CPT and Markowitz, while for the RA type the opposite occurs, no matter the choice of the traditional assets. Commodities create more portfolio turnover only for Markowitz investor and FX worsens turnover for all CPT, Markowitz and LA types. Finally, precious metals induce additional turnover only for the CPT type.

Additionally, we can see that the return-loss measure that takes into account transaction costs, in the case of the real estate is positive for all types. In the second experiment it is negative for CPT and LA, thus the Traditional portfolio has to decrease its return in order to perform equally "well" with the Enhanced. This is also the case, in the third and fourth experiment not only for the CPT and LA types but also for the Risk Averse type.

In the first experiment we find negative opportunity costs θ for CPT, LA and Markowitz and positive for the RA. One needs to give a negative return equal to θ to a CPT, LA and/or Markowitz investor who does not include real estate in her portfolio, so that she becomes willing to include the real state index. On the other hand, one has to give a positive return to the RA investor for the analogous reason. In the commodities experiment we find positive opportunity costs for CPT, LA and Markowitz, while for the RA it is negative. Thus, one has to give a positive return equal to θ to a CPT, LA and/or Markowitz investor in order to make her willing to add commodities in her portfolio and a negative to a RA. In the third experiment the opportunity costs are negative for CPT, LA and Markowitz, while in the last one we find a negative opportunity cost for the Markowitz and RA types. It becomes apparent that, under rationality, no investor of any type and preferences would ever accept an additional negative return in order to include an asset, except if there are other financial reasons whose analysis is away from the scope of this work. The computation of the opportunity cost requires the computation of the expected utility and hence the use of the probability's density function, of the portfolios' returns. Thus, the calculated opportunity cost has taken into account the higher order moments in contrast with the Sharpe ratios.

4. Behavioral Assessment & Conclusions

In our experiments we are interested in unveiling the financial behavior the CPT, LA and Markowitz type of investors will demonstrate. The imprint of their behavior is expressed on the weights of their optimal portfolios, namely the Traditional and the Enhanced. Our benchmark is the RA type of investor, due to its dominant presence in the classical literature of Finance.

Our methodology, besides employing the relevant value and utility functions, it also employs subjective probability transformations. These distortions, together with loss aversion, are the fundamental elements of CPT. In the case of Markowitz, loss aversion is not being used, because it is not provided in the relevant theoretical context, but probability distortion is. Thus, we combine power value/utility functions with subjective probability weighting functions on the cross section of our data. The LA type places a negative coefficient in the losses domain (the same with CPT) in order to declare her aversion. For the benchmark, only the globally concave utility function is being directly elaborated, with no probability distortion. Finally, the risk neutral and the "naive" 1/N behavior are also being incorporated in our analysis.

For both CPT and Markowitz, their essential elements (loss aversion and probability distortion) stem from multiple observed phenomena of choice (resulting from cognitive biases which can be thought as systematic errors) such as narrow framing, overconfidence and mental accounting together with myopic investment decisions. For LA we follow the work of Benartzi and Thaler (BT, 1995), who argue that investors are primarily interested in avoiding losses and when they do, subsequently they research ways to improve their investing performance. Time also plays a crucial role in financial decision-making, especially when it comes to the discretion of a strategy to mature. When there is “time shortage”, effort reasoning is disabled and heuristics take action. Heuristics are decision making shortcuts supported by sentimental factors such as anxiety, herd behavior etc. When there is abundance of time, temperance and prudence can be the main drivers of financial decision-making resulting into more sober choices and/or practices.

In general, short term gambles are supported from risk seeking behaviors (convex part in the value function), while long term gambles from risk averting (concave part in the value function). One way or another, there is strong evidence that loss aversion is always present, making the value function steeper near the origin, in the losses domain. Loss aversion is a documented phenomenon of choice where potential or realized losses loom larger than gains (approx 2 times more). While classical theory in Finance sets the RA type of investor as the norm, CPT and Markowitz remain descriptive. The difference between those two approaches is that the first dictates how investors should make decisions (through effort reasoning) while the later, how they actually do.

In Finance, p.w.f. play a more important role than loss aversion, because it is a risk related field. However, many scholars argue that loss aversion is more than enough in "capturing" real time financial behavior (Barberis and Thaler, 2003). Thus, in our work we employ them both and assess their outcomes. Moreover, it is crucial for the value functions and the p.w.f. to have a similar shape (Levy and Levy 2004), when they are combined. This means that for the CPT case the p.w.f. must be S-shaped while for the Markowitz case reverse S-shaped. Up to our knowledge, there is no other study that uses a similar approach, especially when it comes to set comparisons.

All experiments in our analysis focus on the comparison of the aggregate returns of the Traditional and Enhanced portfolios, for the aforementioned investor types. The Traditional portfolio consists of government bonds, corporate bonds and indices while the Enhanced is in fact the Traditional one augmented with different asset classes, that fall into four main categories which are: precious metals, foreign exchange, commodities and real estate.

4.1 Metals

In this experiment, the portfolio is being enhanced with four precious metals which are gold, platinum, palladium and silver. We calculate their first four central moments and see that they deviate from normality. Thus, our non-parametric approach seems appropriate. The average optimal weights reveal significant differences between all investor types. The first main difference is that the Markowitz type decides to invest heavily on US treasury and corporate bonds, about 90% of the wealth in the Enhanced and 95% in the Traditional portfolio, while CPT and RA do not. Regarding LA we can observe that she places almost all her investment on the S&P500 Energy Index, most probably because of the combination of a relatively high mean and high positive kurtosis, in the Traditional portfolio. Regarding the Enhanced, she places almost all her weight on Silver. The aforementioned assets exhibit moderate average returns and skewness but high kurtosis either negative or positive. Because of their platykurtosis, they are regarded as safe havens and this is how the Markowitz type hedges against market's turbulence. Another significant difference is that the Markowitz type fully diversifies his two portfolios across all assets, while CPT, LA, RN and RA do not, and they exhibit a more concentrated investing strategy. CPT, RN and RA reject US treasury bonds as well as AAA corporate bonds and choose to concentrate on BAA US corporate bonds. These assets have the highest average return, negative skewness and negative kurtosis. Hence, they seem appropriate for risk adverse, or just indifferent preferences. Negative skewness supports attitudes towards long-sighted strategies because they attract investments for the long term and not short-term gamblers. Regarding precious metals, the weights put on by CPT and RA investors are significant higher than Markowitz. This may be due to the fact that both CPT and RA share similar behavioral patterns on gains. They both exhibit risk aversion, with the CPT exhibiting also strong aversion towards losses. Finally for the RN type, by following a non-sophisticated strategy, she selects US BAA Corporate bonds, mostly, because they present the highest average return of all bonds.

Three out of four precious metals demonstrate relatively low positive skewness supporting this way a non-myopic rationale because observations are not significantly asymmetrically distributed around the mean. We can see that the CPT investor chooses to place, among the four metals, the biggest weight on Palladium which has the highest positive skewness. RA chooses a more equally weighted strategy while Markowitz shows little interest on these assets. LA invests almost fully on Silver and S&P500 Energy Index, while RN on US Corp. BAA Bonds. One interesting result is that only the LA demonstrates different distribution of weights between the Traditional and Enhanced portfolios. We believe that the reason behind this outcome is that the presence of solely the loss aversion coefficient can create a significant impact on decision-making.

Two (Palladium and Platinum) out of four precious metals present positive excess kurtosis and the other two negative. For the first two this is an indication for outliers while for the other two (Gold and Silver) exactly the opposite. As we can see in the weights' distribution, Markowitz, CPT and RA investor types place small weights on them. However CPT and RA place more weights on metals that are platykurtic, as expected due to less extreme outliers, while Markowitz places the greatest weight on Gold which is a platykurtic asset with a moderate average return.

The BAA US corporate bond presents the smallest negative skewness, in absolute terms. Both CPT and RA place greater or equal to 90% of their weights on it. While Markowitz decides to place about 70% of his investment on US Treasury bonds and AAA corporate bonds because of positive skewness, which encourages short term gambles that can lead to relatively high returns, on average.

In particular, for the CPT type we can see a 10% difference between (E) and (T) portfolios on the US corporate BAA bond. This is an outcome that can be justified because of the presence of precious metals in the investment universe and hence the investor wants to benefit from them. On the other hand and for the same assets, we can see that the RA type retains a more conservative attitude towards metals and the difference in the US corporate BAA bond is not that high. Despite the fact that both CPT and RA share the same curvature in the utility function on gains, the aforementioned difference comes from the probability weighting functions, which the CPT type employs.

When it comes to Markowitz, it is important to note that between (E) and (T) portfolios there is basically no difference in weights, in total, on US Treasury and Corporate bonds. When precious metals are introduced, what changes is the diversification on other assets. This means that despite the introduction of new assets in the portfolio, the Markowitz type remains stable on his initial selections. Hence, we see that the risk seeking attitude implied by the value function is mitigated from the subjective probability distortion.

4.2 FX

In this experiment, the portfolio contains foreign exchange of eighteen ratios with respect to the US dollar. We calculate their first four central moments and observe that they differ significantly from a normal distribution. Thus, our non-parametric model is appropriate. All these assets exhibit relatively low average returns, moderate skewness but high (excess) kurtosis, in absolute values. More particularly, the Chinese Yuan exhibits extremely high skewness and (excess) kurtosis, implying the presence of extreme outliers. This extremity activates risk aversion for both CPT, RN and RA and thus they select not to invest on it at all, while Markowitz and LA both place a small fraction of wealth, most probably due to the risk seeking feature, for the former. The latter places a negligible weights and this it does not induce any further analysis. The same pattern holds true for the Hong Kong dollar as well as for the Singapore dollar. LA prefers almost exclusively the TAIWAN exchange ratio, which demonstrated high skewness and kurtosis while its average return is one of the lowest compared to other exchange ratios.

Again, Markowitz places almost 95% of his wealth on US Treasury and Corporate bonds and diversifies across all assets. We believe that this full diversification is due to its risk seeking behavior in order not to miss out (Fear of Missing Out cognitive bias) any opportunity. While for the Markowitz type we could say that he is being bold in his decision-making, this is not the case for CPT and RA. Again, these two personae share similar behavior on gains, because of the same shape of their utility functions. What sets them apart is the subjective probability distortion on behalf of the CPT type. This is apparent on their selection regarding the US BAA corporate bond. The RA places significantly much more weight on it, most probably because there is no probability distortion applied, while this distortion discourages the CPT type to invest any further on it.

We can also see that the CPT investor is being riskier in investing in foreign exchange compared to RA. We believe that probability distortion causes this behavior because it tends to overweight small probabilities of high returns. The FX assets, exhibit high values of excess kurtosis, both positive and negative. Thus, opportunities for both short-term and long-term gambles are present, in the CPT investor's mind, because simple descriptive statistics reveals the presence of outliers. It is true that an outlier in the stock market may result into a huge investing opportunity, apart from being a potential total disaster. The rationale for the CPT type in the case of the disaster is that, she decides to gamble (risk seeking) in order to avoid substantial losses. The combination of probability distortion together with loss aversion, initiates this relatively small, although present investment on FX.

4.3 Commodities

In this experiment, the enhanced portfolio contains commodities such as Brent cruel oil, cocoa, corn etc, which are 14 in total. We calculate their first four central moments and see that they deviate significantly from a normal distribution. Thus, our non-parametric approach seems appropriate.

This time, the Markowitz type in both (E) and (T) portfolios, despite diversifying across all assets, places almost all his weight (about 98%) on US Treasure and Corporate bonds. All commodities, except for one, are platykurtic and hence there are no outliers that could result into a potential investing scenario with large returns. Thus, the risk seeking in gains feature, of this type of investor, cannot be triggered.

On the other hand both CPT and RA, who seem to be making analogous decisions, choose not to position themselves on US Treasury and AAA Corporate bonds but only on BAA . They choose to invest on commodities, a considerable fraction of their capital, which is approximately 10% for the CPT type and about 4% for the RA type. This is the case because the RA’s position is again concentrated more on US Corporate BAA bonds by 15%, on average, more than CPT type. Once again, their behaviors mirror due to the curvature of the utility function over gains. Any difference exists because of the probability distortion. The absence of outliers in commodities creates a fertile environment for attitudes averse towards risky gains. Moreover, their concentrated investing strategy in the (E) as well as in the (T) portfolio, is a strong indication of loss aversion. It is a fact that bigger exposure, through broad diversification, besides offering hedging creates at the same time bigger exposure to potential losses. Thus, loss aversion drives the CPT type to have an even more concentrated position than the RA type. Thus, for both CPT and RA, full diversification across all assets is absent and more concentrated positions are present. This absence is declared with a zero weight on an asset. RN again concentrates her strategy on US BAA corp. bonds. Her attitude, as well as the attitude of the LA investor regarding selected assets, remains the same with the previous experiments. It seems that the RN and LA types follow a more conservative path and since they have found a well performing strategy (e.g. choosing traditional asset classes), they are not willing to change it. We could argue that they both exhibit aversion to any alternatives, also known as the "status quo" cognitive bias, the situation where one is not willing to change prior choices given that these choices create satisfactory outcomes.

4.4 Real estate

Finally, in this last experiment, (E) contains the S&P500 Real Estate total return index. We calculate its first four central moments and see that it deviates significantly from the normal distribution. Thus, our non-parametric approach, again, is appropriate. We can see that its distribution is leptokurtic, thus outliers are present and moreover the distribution is positively skewed. This regime is ideal for gambles in the short term with expectations for high payoffs, because of the presence of extreme return values. Usually, investors that go after such kind of potentially profitable situations do not use sophisticated strategies but rather simple heuristics. One simple heuristic could be for example the “quickly enter and quickly exit” a position in order to benefit from a temporal upward price movement. Positively skewed gambles are ideal for this kind of rationale. Moreover, with this particular asset class we can also comment, through hindsight, the housing market crisis of 2008. It is a fact that the housing market bubble, before it burst, attracted a huge number of such kind investors. In bubble schemes, the reference point of the value function for both CPT and Markowitz “moves” together with the bubble expansion. This can create an absurd perception of gains and losses. This absurdness feeds itself as prices skyrocketed and overconfidence of potentially even higher profits contaminated the market. Thus, upon bubble explosion this high overconfidence, and subsequently even higher transactions' volume, is transformed into unprecedented losses.

Once again the Markowitz type concentrates almost entirely on US Treasure and Corporate bonds. The CPT investor decides to place about 9% of his weights on real estate, while the other personas go significantly lower (0.03% and 1%). Basically, the CPT type transfers 10% from US BAA Corporate bonds invested capital to the real estate index, when the latter is introduced in the portfolio. This behavior stems from the fact that the housing market has been traditionally assumed as a safe heaven. It was never expected to fail. Thus, loss aversion drove investing decisions towards taking positions by including real estate, no matter it eventually crashed. Unfortunately, this time the LA investor fails to prevent losses and again by following a concentrated strategy, on the Real Estate index, her payoffs plummet. The RN investor, as before, keeps the same position with the same asset and secures herself from the market's turmoil.

5. Conclusions

In this work, we are interested in examining how different investor personas "behave" when forming optimal portfolios of different asset classes. We are primarily interested in the performance of the three main decision-making personas in Behavioral Finance namely, Cumulative Prospect Theory, Markowitz and Loss Averse type. We also employ the most fundamental personality in Economics and Finance, the Risk Averse type, as well as two "side" personas namely the Risk Neutral and the "naive" investor.

All behaviors, imprinted on portfolio weights, are extracted through in-sample as well as out-of-sample tests. We also employ parametric and non-parametric performance measures. In all cases, these investor types exhibit different attitudes towards risk/uncertainty and thus subsequently optimal portfolios formulations, in-sample as well as out-of-sample. In our analysis we weight more the out-of-sample results because it is a mimicking procedure of forming real time investing strategy of buy-and-hold optimal portfolios.

In all cases we can say that indeed, the specialties of each one investor persona are expressed on portfolios by revealing strong or subtle differences. The CPT investor follows in general a conservative strategy, in all experiments, in order to avoid losses without however succeeding ever time. The Markowitz type succeeds in performing equivalently well in all experiments, with no substantial differences in aggregate returns. Thus we would say that his "financial" attitude is more suitable in forming well-performing portfolios, at least when compared to the CPT type. Our benchmark persona, the RA, also succeeds in performing well in all experiments, in the sense that she does not realize losses and performs more or lees the same (and sometimes better) as the Markowitz type. For the RN, we would say that she is on the same path with RA, while the LA type (or the LA heuristic) is able to attain returns no other persona can. Hence, we could sparingly state that when it comes to aggregate returns (or cumulative portfolio performance), the LA heuristic (persona) dominates. Finally, the naive investor performs poorly in three out of four experiments.

6. Appendix

Numerical Implementation and Computational Strategy

6.1 Cumulative Prospect Theory investor type

For the negative part, we have a set of convex utility functions of the form:

$$v(u) = \sum_{n=1}^{n_1} \pi^{-}(w_n) \max(u, z_n)$$

We define

$$c_{0,n} := \sum_{m=n}^{n_1} (c_{1,m+1} - c_{1,m}) z_m;$$
$$c_{1,n} := \sum_{m=1}^{n} w_m;$$
$$\mathcal{N} := \{n = 1, \cdots, n_1 : w_n > 0\} \bigcup \{n_1\}.$$

Below, we give the mathematical formulation for the first optimization problem $\sup_{\lambda\in\Lambda} E_{F_N}[u(\lambda^T X)]$], that yields the optimal portfolio λ. The same formulation is used for the second optimization $\sup_{\kappa\in K} E_{F_N}[u(\kappa^T Y)]$.

For any given $u \in V_{-}$ $\sup_{\lambda\in\Lambda} E_{F_N}[u(\lambda^T X)]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{6}$$

s.t.

$$y_t \leq \lambda^T X_t c_{1,p} + Q_t^- + Q_t^+, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$y_t \leq c_{0,p} + Q_t^- + Q_t^+, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$Q_t^- \geq c_{0,p} - \lambda^T X_t c_{1,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$y_t \leq \lambda^T X_t c_{1,p} + c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$Q_t^+ \geq \lambda^T X_t c_{1,p} - c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$

$$\sum_{i=1}^{M} \lambda_i = 1;$$

$$\lambda_i \geq 0, : i = 1, \cdots, M;$$

$$Q_t^-, Q_t^+, \geq 0, : t = 1, \cdots, T;$$

$$y_t : \text{free}, : t = 1, \cdots, T.$$

For the positive part, we have a set of concave utility functions of the form:

$$v(u) = \sum_{p=1}^{p_1} \pi^+(w_p) \min(u, z_p)$$

We define:

$$c_{0,p} := \sum_{m=p}^{p_1} (c_{1,m+1} - c_{1,m}) z_m;$$

$$c_{1,p} := \sum_{m=p}^{p_1} w_m;$$

$$\mathcal{P} := \{p = 1, \cdots, p_1 : w_p > 0\} \bigcup \{p_1\}.$$

For any given $u \in V_+$ $\sup_{\lambda \in \Lambda} E_{F_N}[u(\lambda^T X)]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{7}$$

s.t.

$$y_t \leq \lambda^T X_t c_{1,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$

$$y_t \leq c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$

$$\sum_{i=1}^{M} \lambda_i = 1;$$

$$\lambda_i \geq 0, : i = 1, \cdots, M;$$

$$y_t : \text{free}, : t = 1, \cdots, T.$$

6.2 Markowitz investor type

For the negative part, we have a set of concave utility functions of the form:

$$v(u) = \sum_{n=1}^{n_1} \pi^-(w_n) \min(u, z_n)$$

We define:

$$c_{0,n} := \sum_{m=n}^{n_1} (c_{1,m+1} - c_{1,m}) z_m;$$

$$c_{1,n} := \sum_{m=1}^{n} w_m;$$

$$\mathcal{N} := \{n = 1, \cdots, n_1 : w_n > 0\} \bigcup \{n_1\}.$$

Below, we give the mathematical formulation for the first optimization problem $\sup_{\lambda\in\Lambda} E_{F_N}[u(\lambda^T X)]$], that yields the optimal portfolio λ. The same formulation is used for the second optimization $\sup_{\kappa\in K} E_{F_N}[u(\kappa^T Y)]$.

For any given $u \in V_-$ $\sup_{\lambda\in\Lambda} E_{F_N}[u(\lambda^T X)]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{8}$$

s.t.

$$y_t \leq \lambda^T X_t c_{1,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$

$$y_t \leq c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$

$$\sum_{i=1}^{M} \lambda_i = 1;$$

$$\lambda_i \geq 0, : i = 1, \cdots, M;$$

$$y_t : \text{free}, : t = 1, \cdots, T.$$

For the positive part, we have a set of convex utility functions of the form:

$$v(u) = \sum_{p=1}^{p_1} \pi^+(w_p) \max(u, z_p)$$

We define:

$$c_{0,p} := \sum_{m=p}^{p_1} \left(c_{1,m+1} - c_{1,m}\right) z_m;$$

$$c_{1,p} := \sum_{m=p}^{p_1} w_m;$$

$$\mathcal{P} := \{p = 1, \cdots, p_1 : w_p > 0\} \bigcup \{p_1\}.$$

For any given $u \in V_+$ $\sup_{\lambda \in \Lambda} E_{F_N}[u(\lambda^T X)]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{9}$$

s.t.

$$y_t \leq \lambda^T X_t c_{1,p} + Q_t^- + Q_t^+, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$y_t \leq c_{0,p} + Q_t^- + Q_t^+, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$Q_t^- \geq c_{0,p} - \lambda^T X_t c_{1,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$y_t \leq \lambda^T X_t c_{1,p} + c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$Q_t^+ \geq \lambda^T X_t c_{1,p} - c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P};$$
$$\sum_{i=1}^{M} \lambda_i = 1;$$
$$\lambda_i \geq 0, : i = 1, \cdots, M;$$
$$Q_t^-, Q_t^+, \geq 0, : t = 1, \cdots, T;$$
$$y_t : \text{free}, : t = 1, \cdots, T.$$

6.3 Risk neutral investor type

For optimal portfolios, the Risk neutral type maximizes:

$$\max T^{-1} \sum_{t=1}^{T} Y_t \tag{10}$$

s.t.

$$M(b_t - 1) \leq \lambda^T X_t \leq M(b_t), : t = 1, \cdots, T \tag{11}$$

$$-M(F_t) \leq L_t \leq M(F_t), : t = 1, \cdots, T \tag{12}$$

$$M(F_t - 1) \leq W_t \leq M(1 - F_t), : t = 1, \cdots, T \tag{13}$$

$$Z_t = -(1 - 2b_t) \cdot \lambda^T X_t, : t = 1, \cdots, T \tag{14}$$

$$M(F_t - 1) \leq L_t - Z_t \leq M(1 - F_t), : t = 1, \cdots, T \tag{15}$$

$$-M(F_t) \leq W_t - Z_t \leq M(F_t), : t = 1, \cdots, T \tag{16}$$

$$Y_t = L_t b_t + W_t(1 - b_t), : t = 1, \cdots, T \tag{17}$$

$$\sum_{i=1}^{M} \lambda_i = 1; \tag{18}$$

$$\lambda_i \geq 0, : i = 1, \cdots, M \tag{19}$$

$$F_t, b_t \in \{0, 1\}, : t = 1, \cdots, T \tag{20}$$

$$Y_t : \text{free}, : t = 1, \cdots, T$$

$$M \in \mathbb{Z}_{++}$$

6.4 Loss Averse investor type

For optimal portfolios, the Loss Averse type maximizes:

$$\max T^{-1} \sum_{t=1}^{T} Y_t \tag{21}$$

s.t.

$$M(b_t - 1) \leq \lambda^T X_t \leq M(b_t); t = 1, \cdots, T \tag{22}$$

$$-M(F_t) \leq L_t \leq M(F_t); t = 1, \cdots, T \tag{23}$$

$$M(F_t - 1) \leq W_t \leq M(1 - F_t); t = 1, \cdots, T \tag{24}$$

$$Z_t = \left[-(1 - 2b_t) \cdot \lambda^T X_t \right]; t = 1, \cdots, T \tag{25}$$

$$M(F_t - 1) \leq L_t - Z_t \leq M(1 - F_t); t = 1, \cdots, T \tag{26}$$

$$-M(F_t) \leq W_t - v \cdot Z_t \leq M(F_t); t = 1, \cdots, T \tag{27}$$

$$Y_t = L_t b_t + W_t(1 - b_t); t = 1, \cdots, T \tag{28}$$

$$\sum_{i=1}^{M} \lambda_i = 1; \tag{29}$$

$$\lambda_i \geq 0; i = 1, \cdots, M \tag{30}$$

$$F_t, b_t \in \{0, 1\}; t = 1, \cdots, T \tag{31}$$

$$Y_t : \text{free}; t = 1, \cdots, T$$

$$v \geq 0; \textit{loss aversion coefficient}$$

$$M \in \mathbb{Z}_{++}$$

6.5 Risk averse investor type

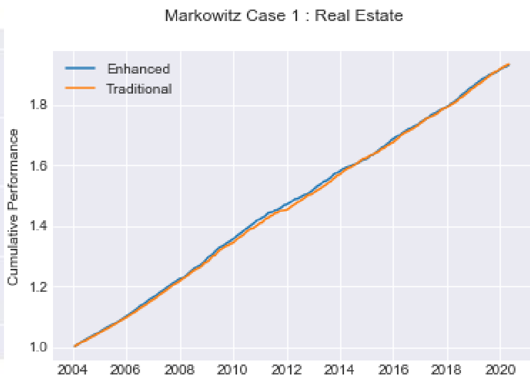
For any given $u \in V_{concave}$ $\sup_{\lambda \in \Lambda} E_{F_N}[u(\lambda^T X)]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{32}$$

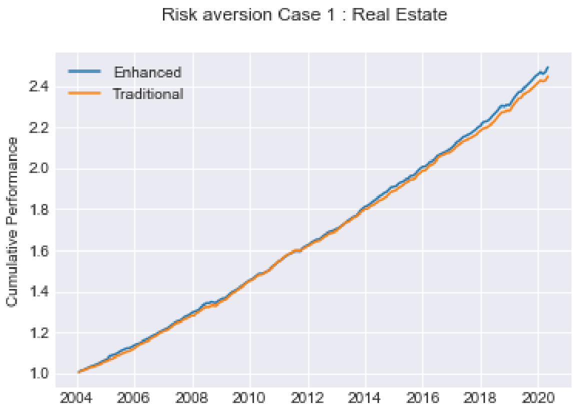
s.t.

$$\begin{aligned} & y_t \leq \lambda^T X_t c_{1,p} + c_{0,p}, : t = 1, \cdots, T; p \in \mathcal{P}; \\ & \sum_{i=1}^{M} \lambda_i = 1; \\ & \lambda_i \geq 0, : i = 1, \cdots, M; \\ & y_t : \text{free}, : t = 1, \cdots, T. \end{aligned} \tag{33}$$

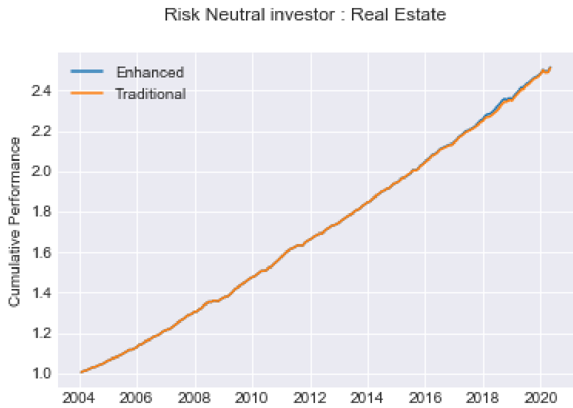
(a) CPT type of investor



(b) Markowitz type of investor

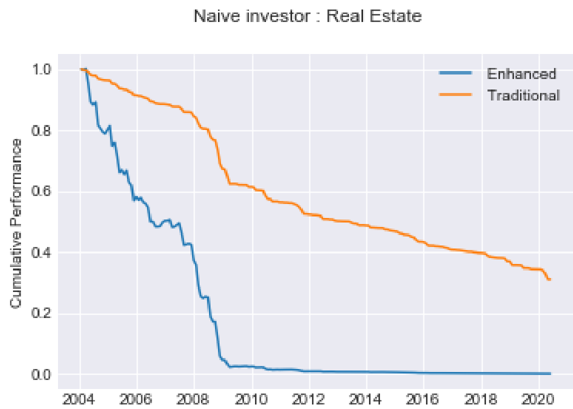


(c) Risk averse type of investor



(d) Risk neutral type of investor

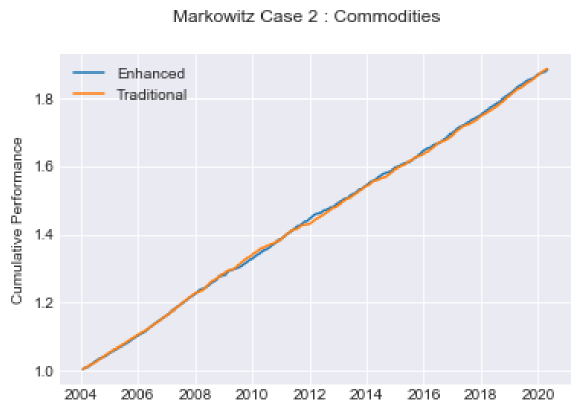
(f) Loss averse type of investor



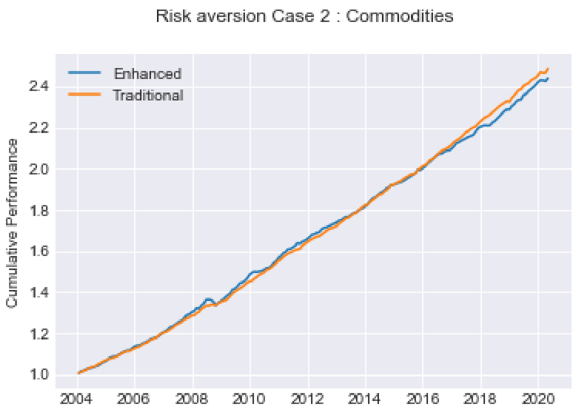
(f) Naive type of investor

Figure 1: Cumulative performance in the first case of real estate

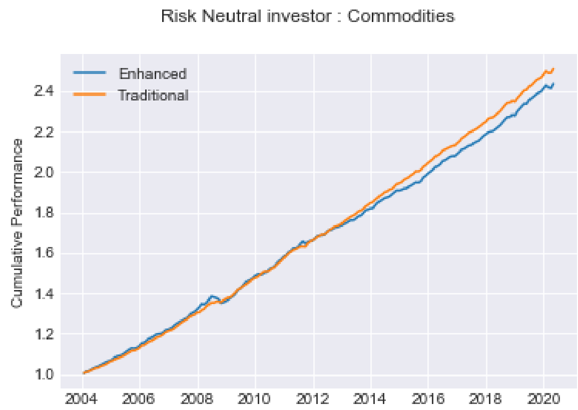
(a) CPT type of investor



(b) Markowitz type of investor



(c) Risk averse type of investor

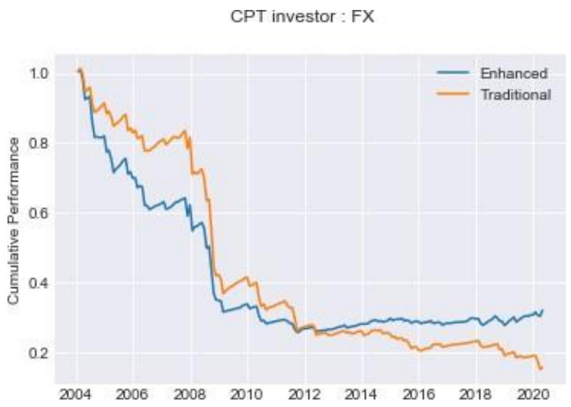


(d) Risk neutral type of investor

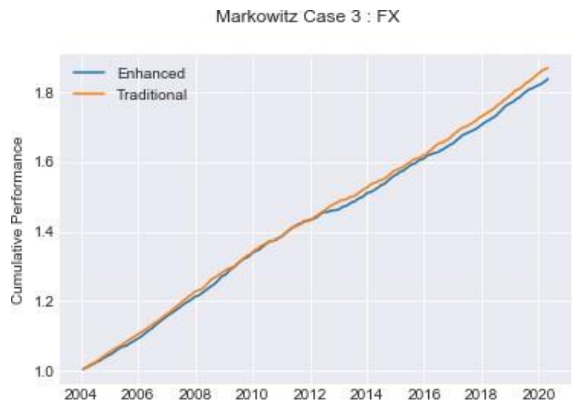
(f) Loss averse type of investor

(f) Naive type of investor

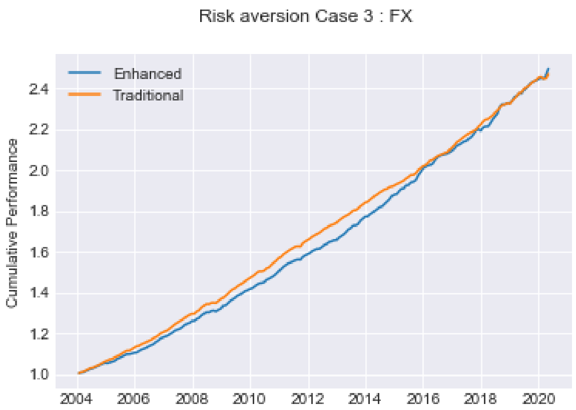
Figure 2: Cumulative performance in the second case of commodities



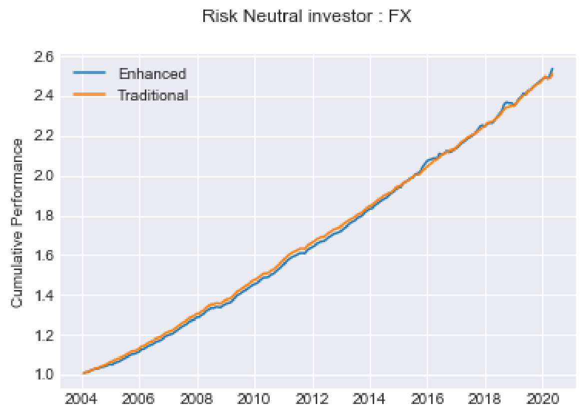
(a) CPT type of investor



(b) Markowitz type of investor

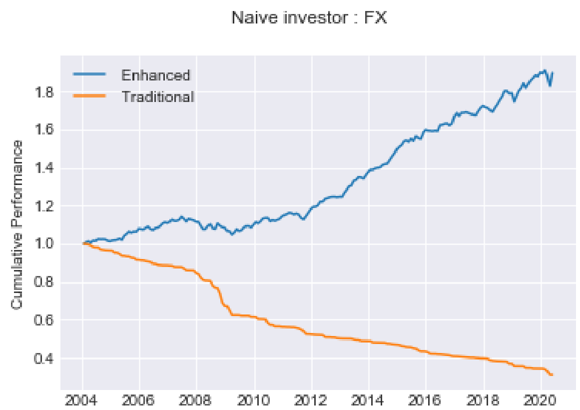


(c) Risk averse type of investor



(d) Risk neutral type of investor

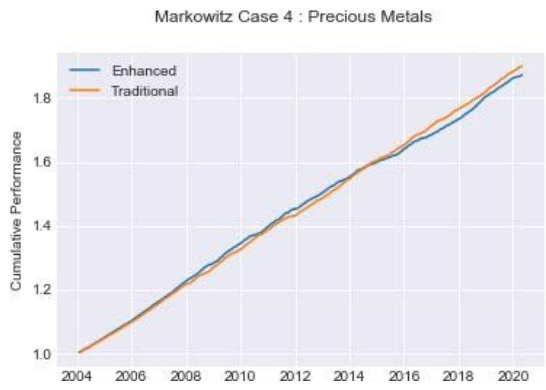
(e) Loss averse type of investor



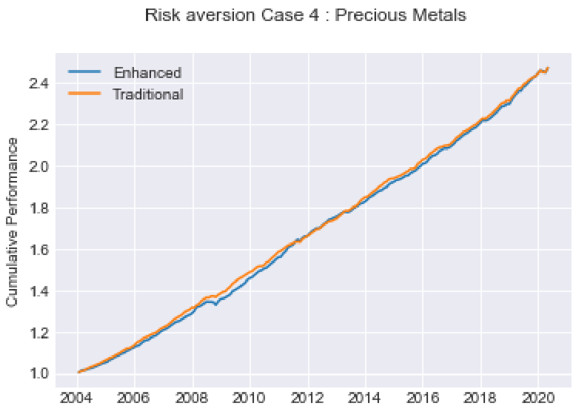
(f) Naive type of investor

Figure 3: Cumulative performance in the third case of FX

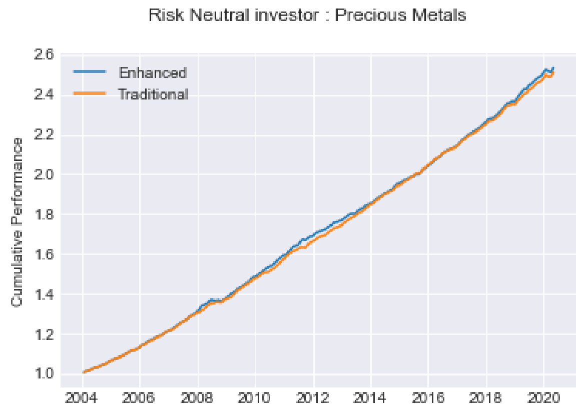
(a) CPT type of investor



(b) Markowitz type of investor

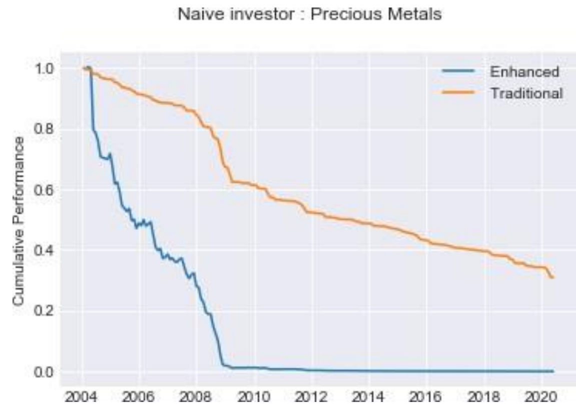


(c) Risk averse type of investor



(d) Risk neutral type of investor

(e) Loss averse type of investor



(f) Naive type of investor

Figure 4: Cumulative performance in the fourth case of precious metals

Table 1: Descriptive Statistics (whole sample)

Entries report the descriptive statistics on monthly returns for the alternative asset classes that proxies the benchmark ("traditional") asset universe and the enhanced with commodities, foreign exchange (FX), real estate and precious metals. Data spans the period 31/1/1994 - 30/4/2020

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
US TREASURY CONST MAT 30 YEAR (D) - MIDDLE RATE	0,0037	0,0012	0,1382	-0,8069
US TREASURY CONST MAT 5 YEAR (D) - MIDDLE RATE	0,0028	0,0016	0,2957	-1,1545
US CORP BONDS MOODYS SEASONED AAA - MIDDLE RATE	0,0044	0,0012	0,1141	-1,0839
US CORP BONDS MOODYS SEASONED BAA - MIDDLE RATE	0,0052	0,0011	-0,0299	-1,1719
Bloomberg Barclays U.S. Aggregate USD - Average price	0,0001	0,0104	-0,2643	1,0371
S&P500 ES HEALTH CARE - TOT RETURN IND	0,0105	0,0431	-0,3378	0,3660
S&P500 ES CONSUMER DISCRETIONARY - TOT RETURN IND	0,0093	0,0516	-0,1809	1,6516
S&P500 ES CONSUMER STAPLES - TOT RETURN IND	0,0086	0,0362	-0,4163	1,4993
S&P500 ES INDUSTRIALS - TOT RETURN IND	0,0083	0,0513	-0,5748	1,8640
S&P500 ES INFO TECHNOLOGY - TOT RETURN IND	0,0127	0,0719	-0,3438	1,2388
S&P500 ES MATERIALS - TOT RETURN IND	0,0074	0,0586	-0,0399	1,5167
S&P500 ES COMM. SVS - TOT RETURN IND	0,0061	0,0559	0,2878	3,2960
S&P500 ES UTILITIES - TOT RETURN IND	0,0072	0,0439	-0,5118	0,7757
S&P500 ES FINANCIALS - TOT RETURN IND	0,0078	0,0622	-0,6713	3,0048
S&P500 ES ENERGY - TOT RETURN IND	0,0074	0,0618	-0,3933	4,6310
ICE-BRENT CRUDE OIL TRc1 - SETT. PRICE	0,0068	0,0945	-0,6220	4,4480
CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0018	0,0509	-0,4903	1,9342
CME-FEEDER CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0021	0,0444	-0,3445	1,5193
CME-LEAN HOGS COMP. CONTINUOUS - SETT. PRICE	0,0071	0,1118	0,2235	1,1028
CBT-CORN COMP. CONTINUOUS - SETT. PRICE	0,0035	0,0821	-0,1645	0,9930
CBT-SOYBEANS COMP. CONT. - SETT. PRICE	0,0033	0,0731	-0,4592	1,4217
CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE	0,0049	0,0887	0,5980	1,6038
Crude Oil WTI NYMEX Close M23 U\$/BBL	0,0056	0,0902	-0,4835	2,6954
CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE	0,0056	0,0451	0,1733	1,2381
CMX-SILVER 5000 OZ CONTINUOUS - SETT. PRICE	0,0067	0,0818	0,0906	0,8345
CSCE-COTTON #2 CONTINUOUS - SETT. PRICE	0,0034	0,0874	-0,1947	1,2866
CSCE-COFFEE 'C' CONTINUOUS - SETT. PRICE	0,0067	0,1079	1,0447	2,7274

Table 2: Descriptive Statistics (whole sample) continued

Entries report the descriptive statistics on monthly returns for the alternative asset classes that proxies the benchmark ("traditional") asset universe and the enhanced with commodities, foreign exchange (FX), real estate and precious metals. Data spans the period 31/1/1994 - 30/4/2020

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
CSCE-COCOA CONTINUOUS - SETT. PRICE	0,0063	0,0888	0,3563	0,9885
CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE	0,0042	0,0940	0,3341	0,7742
S&P UNITED STATES REIT U\$ - TOT RETURN IND	0,0004	0,0239	0,7920	5,0453
CANADIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0239	0,7920	5,0453
DANISH KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0273	0,3156	1,1177
JAPANESE YEN TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0302	-0,2430	2,4272
NORWEGIAN KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0014	0,0309	0,4380	1,2649
SOUTH AFRICA RAND TO US \$ NOON NY - EXCHANGE RATE	0,0064	0,0450	0,6399	1,3514
SWEDISH KRONA TO US NOON NY - EXCHANGE RATE	0,0010	0,0305	0,1646	0,5371
SWISS FRANC TO US \$ NOON NY - EXCHANGE RATE	-0,0009	0,0291	-0,0003	1,5719
AUSTRALIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0007	0,0338	0,8349	3,8874
NEW ZEALAND \$ TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0351	0,5707	1,9608
UK £ TO US \$ NOON NY - EXCHANGE RATE	0,0008	0,0243	0,5336	1,7642
INDIAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0030	0,0199	0,7235	3,5632
SRI LANKAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0044	0,0136	1,4274	8,5387
CHINESE YUAN TO US \$ NOON NY - EXCHANGE RATE	0,0009	0,0290	16,2481	279,9218
HONG KONG \$ TO US \$ NOON NY - EXCHANGE RATE	0,0000	0,0013	-0,8130	6,3969
SINGAPORE \$ TO US \$ NOON NY - EXCHANGE RATE	-0,0003	0,0164	0,5122	2,8569
THAI BAHT TO US \$ NOON NY - EXCHANGE RATE	0,0012	0,0310	2,4579	28,4361
SOUTH KOREAN WON TO US \$ NOON NY - EXCHANGE RATE	0,0021	0,0423	4,0250	41,7343
TAIWAN NEW \$ TO US \$ NOON NY - EXCHANGE RATE	0,0005	0,0155	0,3397	4,6853
MEXICAN PESO TO US \$ NOON NY - EXCHANGE RATE	3,3330	55,7453	17,6651	313,2405
Palladium U\$/Troy Ounce	0,0139	0,1015	0,4711	3,1720
London Platinum Free Market \$/Troy oz	0,0041	0,0617	-0,5432	3,1083
Gold, Handy & Harman Base \$/Troy Oz	0,0057	0,0449	0,1398	1,2947
Silver, Handy&Harman (NY) U\$/Troy OZ	0,0067	0,0804	0,1090	0,9638

Table 3: Descriptive Statistics

Entries report the descriptive statistics on monthly returns for the alternative asset classes. Data spans the period 1/1/2000 - 4/30/2020.

	AVERAGE	ST.DEV	SKEWNESS	KURTOSIS
US TREASURY CONST MAT 30 YEAR (D) - MIDDLE RATE	0,0037	0,0012	0,1382	-0,8069
US TREASURY CONST MAT 5 YEAR (D) - MIDDLE RATE	0,0028	0,0016	0,2957	-11,545
US CORP BONDS MOODYS SEASONED AAA - MIDDLE RATE	0,0044	0,0012	0,1141	-10,839
US CORP BONDS MOODYS SEASONED BAA - MIDDLE RATE	0,0052	0,0011	-0,0299	-11,719
Bloomberg Barclays U.S. Aggregate USD - Average price	0,0001	0,0104	-0,2643	10,371
S&P500 ES HEALTH CARE - TOT RETURN IND	0,0105	0,0431	-0,3378	0,3660
S&P500 ES CONSUMER DISCRETIONARY - TOT RETURN IND	0,0093	0,0516	-0,1809	16,516
S&P500 ES CONSUMER STAPLES - TOT RETURN IND	0,0086	0,0362	-0,4163	14,993
S&P500 ES INDUSTRIALS - TOT RETURN IND	0,0083	0,0513	-0,5748	18,640
S&P500 ES INFO TECHNOLOGY - TOT RETURN IND	0,0127	0,0719	-0,3438	12,388
S&P500 ES MATERIALS - TOT RETURN IND	0,0074	0,0586	-0,0399	15,167
S&P500 ES COMM. SVS - TOT RETURN IND	0,0061	0,0559	0,2878	32,960
S&P500 ES UTILITIES - TOT RETURN IND	0,0072	0,0439	-0,5118	0,7757
S&P500 ES FINANCIALS - TOT RETURN IND	0,0078	0,0622	-0,6713	30,048
S&P500 ES ENERGY - TOT RETURN IND	0,0074	0,0618	-0,3933	46,310
ICE-BRENT CRUDE OIL TRc1 - SETT. PRICE	0,0068	0,0945	-0,6220	44,480
CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0018	0,0509	-0,4903	19,342
CME-FEEDER CATTLE COMP. CONTINUOUS - SETT. PRICE	0,0021	0,0444	-0,3445	15,193
CME-LEAN HOGS COMP. CONTINUOUS - SETT. PRICE	0,0071	0,1118	0,2235	11,028
CBT-CORN COMP. CONTINUOUS - SETT. PRICE	0,0035	0,0821	-0,1645	0,9930
CBT-SOYBEANS COMP. CONT. - SETT. PRICE	0,0033	0,0731	-0,4592	14,217
CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE	0,0049	0,0887	0,5980	16,038
Crude Oil WTI NYMEX Close M23 U\$/BBL	0,0056	0,0902	-0,4835	26,954
CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE	0,0056	0,0451	0,1733	12,381
CMX-SILVER 5000 OZ CONTINUOUS - SETT. PRICE	0,0067	0,0818	0,0906	0,8345
CSCE-COTTON #2 CONTINUOUS - SETT. PRICE	0,0034	0,0874	-0,1947	12,866
CSCE-COFFEE 'C' CONTINUOUS - SETT. PRICE	0,0067	0,1079	10,447	27,274
CSCE-COCOA CONTINUOUS - SETT. PRICE	0,0063	0,0888	0,3563	0,9885
CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE	0,0042	0,0940	0,3341	0,7742
S&P UNITED STATES REIT U\$ - TOT RETURN IND	0,0004	0,0239	0,7920	50,453
CANADIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0239	0,7920	50,453
DANISH KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0004	0,0273	0,3156	11,177
JAPANESE YEN TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0302	-0,2430	24,272
NORWEGIAN KRONE TO US \$ NOON NY - EXCHANGE RATE	0,0014	0,0309	0,4380	12,649
SOUTH AFRICA RAND TO US \$ NOON NY - EXCHANGE RATE	0,0064	0,0450	0,6399	13,514
SWEDISH KRONA TO US NOON NY - EXCHANGE RATE	0,0010	0,0305	0,1646	0,5371
SWISS FRANC TO US \$ NOON NY - EXCHANGE RATE	-0,0009	0,0291	-0,0003	15,719
AUSTRALIAN \$ TO US \$ NOON NY - EXCHANGE RATE	0,0007	0,0338	0,8349	38,874
NEW ZEALAND \$ TO US \$ NOON NY - EXCHANGE RATE	0,0003	0,0351	0,5707	19,608
UK £ TO US \$ NOON NY - EXCHANGE RATE	0,0008	0,0243	0,5336	17,642
INDIAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0030	0,0199	0,7235	35,632
SRI LANKAN RUPEE TO US \$ NOON NY - EXCHANGE RATE	0,0044	0,0136	14,274	85,387
CHINESE YUAN TO US \$ NOON NY - EXCHANGE RATE	0,0009	0,0290	162,481	2,799,218
HONG KONG \$ TO US \$ NOON NY - EXCHANGE RATE	0,0000	0,0013	-0,8130	63,969
SINGAPORE \$ TO US \$ NOON NY - EXCHANGE RATE	-0,0003	0,0164	0,5122	28,569
THAI BAHT TO US \$ NOON NY - EXCHANGE RATE	0,0012	0,0310	24,579	284,361
SOUTH KOREAN WON TO US \$ NOON NY - EXCHANGE RATE	0,0021	0,0423	40,250	417,343
TAIWAN NEW \$ TO US \$ NOON NY - EXCHANGE RATE	0,0005	0,0155	0,3397	46,853
MEXICAN PESO TO US \$ NOON NY - EXCHANGE RATE	33,330	557,453	176,651	3,132,405
Palladium U\$/Troy Ounce	0,0139	0,1015	0,4711	31,720
London Platinum Free Market \$/Troy oz	0,0041	0,0617	-0,5432	31,083
Gold, Handy & Harman Base \$/Troy Oz	0,0057	0,0449	0,1398	12,947
Silver, Handy&Harman (NY) U\$/Troy OZ	0,0067	0,0804	0,1090	0,9638

Table 4: Out-of-sample portfolio composition for real estate

Entries report the optimal weights of the traditional portfolio and the augmented portfolio with real estate assets. There are two portfolio types, namely the "traditional" and the augmented. The dataset spans the period 1/1/2000 - 4/30/2020, for a total of 224 monthly returns and the rolling window analysis covers 120 months (10 years).

	cpt augm	cpt trad	markowitz augm	markowitz trad	ra augm	ra trad	la augm	la trad	rn augm
US TREASURY CONST MAT 30 YEAR	0.0000	0.0000	0.2190	0.2802	0.0000	0.0000	0.0001	0.0000	0.0000
US TREASURY CONST MAT 5 YEAR	0.0000	0.0000	0.2333	0.2294	0.0000	0.0000	0.0001	0.0000	0.0000
US CORP BONDS MOODYS SEASONED AAA	0.0000	0.0000	0.3003	0.1833	0.0000	0.0000	0.0002	0.0001	0.0000
US CORP BONDS MOODYS SEASONED BAA	0.7922	0.8801	0.2491	0.3053	0.9370	0.9501	0.0002	0.0001	0.9384
Bloomberg Barclays U.S. Aggregate USD	0.0138	0.0307	0.0011	0.0011	0.0000	0.0000	0.0002	0.0001	0.0000
S&P500 ES HEALTH CARE	0.0027	0.0079	0.0001	0.0001	0.0089	0.0082	0.0002	0.0000	0.0068
S&P500 ES CONSUMER DISCRETIONARY	0.0033	0.0008	0.0001	0.0001	0.0050	0.0026	0.0002	0.0001	0.0043
S&P500 ES CONSUMER STAPLES	0.0162	0.0072	0.0002	0.0003	0.0081	0.0090	0.0003	0.0001	0.0121
S&P500 ES INDUSTRIALS	0.0011	0.0000	0.0001	0.0001	0.0009	0.0011	0.0003	0.0001	0.0001
S&P500 ES INFO TECHNOLOGY	0.0135	0.0079	0.0000	0.0000	0.0052	0.0051	0.0004	0.0001	0.0032
S&P500 ES MATERIALS	0.0074	0.0061	0.0001	0.0001	0.0021	0.0017	0.0005	0.0002	0.0003
S&P500 ES COMM. SVS	0.0097	0.0153	0.0001	0.0001	0.0022	0.0025	0.0006	0.0005	0.0010
S&P500 ES UTILITIES	0.0114	0.0136	0.0001	0.0000	0.0081	0.0072	0.0008	0.0007	0.0057
S&P500 ES FINANCIALS	0.0140	0.0098	0.0000	0.0000	0.0008	0.0002	0.0011	0.0010	0.0000
S&P500 ES ENERGY	0.0277	0.0205	0.0000	0.0001	0.0122	0.0124	0.0018	0.9968	0.0223
S&P UNITED STATES REIT U$	0.0869		0.0004		0.0096		0.9929		0.0059

Table 5: Out-of-sample portfolio composition for commodities

Entries report the optimal weights of the traditional portfolio and the augmented portfolio with commodity assets. There are two portfolio types, namely the "traditional" and the augmented. The dataset spans the period 1/1/2000 - 4/30/2020, for a total of 224 monthly returns and the rolling window analysis covers 120 months (10 years).

	cpt augm	cpt trad	markowitz augm	markowitz trad	ra augm	ra trad	la augm	la trad	rn augm	rn trad
US TREASURY CONST MAT 30 YEAR	0.0000	0.0000	0.2291	0.2650	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000
US TREASURY CONST MAT 5 YEAR	0.0000	0.0000	0.2701	0.2753	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000
US CORP BONDS MOODYS SEASONED AAA	0.0000	0.0000	0.2494	0.1936	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000
US CORP BONDS MOODYS SEASONED BAA	0.7749	0.7185	0.2493	0.2647	0.9270	0.9505	0.0001	0.0002	0.9226	0.9468
Bloomberg Barclays U.S. Aggregate USD	0.0000	0.0934	0.0014	0.0007	0.0000	0.0000	0.0001	0.0002	0.0000	0.0000
S&P500 ES HEALTH CARE	0.0032	0.0156	0.0001	0.0001	0.0057	0.0072	0.0001	0.0002	0.0077	0.0097
S&P500 ES CONSUMER DISCRETIONARY	0.0077	0.0060	0.0000	0.0001	0.0028	0.0037	0.0001	0.0002	0.0030	0.0019
S&P500 ES CONSUMER STAPLES	0.0199	0.0177	0.0001	0.0002	0.0033	0.0086	0.0002	0.0002	0.0111	0.0100
S&P500 ES INDUSTRIALS	0.0000	0.0025	0.0001	0.0001	0.0004	0.0004	0.0002	0.0002	0.0002	0.0001
S&P500 ES INFO TECHNOLOGY	0.0057	0.0133	0.0000	0.0000	0.0034	0.0046	0.0002	0.0002	0.0026	0.0018
S&P500 ES MATERIALS	0.0004	0.0149	0.0000	0.0000	0.0011	0.0023	0.0002	0.0004	0.0001	0.0003
S&P500 ES COMM. SVS	0.0157	0.0279	0.0000	0.0000	0.0031	0.0031	0.0003	0.0007	0.0025	0.0009
S&P500 ES UTILITIES	0.0224	0.0294	0.0000	0.0000	0.0070	0.0052	0.0003	0.0012	0.0074	0.0062
S&P500 ES FINANCIALS	0.0031	0.0228	0.0000	0.0000	0.0002	0.0002	0.0003	0.0021	0.0004	0.0000
S&P500 ES ENERGY	0.0002	0.0381	0.0000	0.0000	0.0029	0.0141	0.0003	0.9942	0.0035	0.0223
ICE-BRENT CRUDE OIL TRc1	0.0107		0.0000		0.0046		0.0003		0.0092	
CME-LIVE CATTLE COMP. CONT	0.0096		0.0000		0.0032		0.0003		0.0004	
CME-FEEDER CATTLE COMP. CONT	0.0042		0.0001		0.0055		0.0003		0.0040	
CME-LEAN HOGS COMP. CONT	0.0215		0.0000		0.0024		0.0004		0.0004	
CBT-CORN COMP. CONT	0.0098		0.0000		0.0015		0.0004		0.0011	
CBT-SOYBEANS COMP. CONT	0.0061		0.0000		0.0015		0.0004		0.0002	
CBT-WHEAT COMPOSITE FUTURES CONT	0.0070		0.0000		0.0019		0.0005		0.0003	
Crude Oil WTI NYMEX Close M+1 U$/BBL	0.0038		0.0000		0.0005		0.0005		0.0001	
CMX-GOLD 100 OZ CONT	0.0151		0.0001		0.0056		0.0006		0.0072	
CMX-SILVER 5000 OZ CONT	0.0074		0.0000		0.0018		0.0007		0.0024	
CSCE-COTTON #2 CONT	0.0058		0.0000		0.0009		0.0012		0.0002	
CSCE-COFFEE 'C' CONT	0.0075		0.0000		0.0028		0.0020		0.0005	
CSCE-COCOA CONT	0.0220		0.0000		0.0047		0.0067		0.0022	
CSCE-SUGAR #11 CONT	0.0166		0.0000		0.0060		0.9832		0.0108	

Table 6: Out-of-sample portfolio composition for foreign exchange rates

Entries report the optimal weights of the traditional portfolio and the augmented portfolio with foreign exchange rates. There are two portfolio types, namely the "traditional" and the augmented. The dataset spans the period 1/1/2000 - 4/30/2020, for a total of 224 monthly returns and the rolling window analysis covers 120 months (10 years).

	cpt augm	cpt trad	markowitz augm	markowitz trad	ra augm	ra trad	la augm	la trad	rn augm	rn trad
US TREASURY CONST MAT 30 YEAR	0.0000	0.0000	0.2763	0.2651	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
US TREASURY CONST MAT 5 YEAR	0.0000	0.0000	0.2186	0.2754	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
US CORP BONDS MOODYS SEASONED AAA	0.0000	0.0000	0.2350	0.2089	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
US CORP BONDS MOODYS SEASONED BAA	0.4986	0.6756	0.2506	0.2495	0.8733	0.9506	0.0000	0.0000	0.8870	0.9468
Bloomberg Barclays U.S. Aggregate USD	0.0237	0.1073	0.0007	0.0006	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
S&P500 ES HEALTH CARE	0.0197	0.0201	0.0001	0.0001	0.0124	0.0092	0.0001	0.0000	0.0147	0.0097
S&P500 ES CONSUMER DISCRETIONARY	0.0011	0.0085	0.0000	0.0000	0.0056	0.0027	0.0001	0.0000	0.0032	0.0019
S&P500 ES CONSUMER STAPLES	0.0214	0.0209	0.0002	0.0002	0.0104	0.0085	0.0001	0.0000	0.0066	0.0100
S&P500 ES INDUSTRIALS	0.0087	0.0037	0.0000	0.0001	0.0007	0.0002	0.0001	0.0000	0.0001	0.0001
S&P500 ES INFO TECHNOLOGY	0.0379	0.0155	0.0000	0.0000	0.0079	0.0043	0.0001	0.0001	0.0140	0.0018
S&P500 ES MATERIALS	0.0194	0.0134	0.0000	0.0000	0.0038	0.0017	0.0001	0.0006	0.0005	0.0003
S&P500 ES COMM. SVS	0.0128	0.0315	0.0000	0.0000	0.0033	0.0027	0.0001	0.3223	0.0020	0.0009
S&P500 ES UTILITIES	0.0184	0.0324	0.0000	0.0000	0.0052	0.0062	0.0001	0.0015	0.0017	0.0062
S&P500 ES FINANCIALS	0.0126	0.0261	0.0000	0.0000	0.0018	0.0002	0.0001	0.0023	0.0000	0.0000
S&P500 ES ENERGY	0.0413	0.0449	0.0000	0.0000	0.0121	0.0136	0.0001	0.6731	0.0230	0.0223
CANADIAN *TOUS* NOON NY	0.0301		0.0001		0.0008		0.0001		0.0000	
DANISH KRONE TO US $ NOON NY	0.0127		0.0000		0.0000		0.0001		0.0000	
JAPANESE YEN TO US $ NOON NY	0.0350		0.0001		0.0012		0.0001		0.0001	
NORWEGIAN KRONE TO US $ NOON NY	0.0112		0.0000		0.0028		0.0001		0.0014	
SOUTH AFRICA RAND TO US $ NOON NY	0.0615		0.0000		0.0226		0.0001		0.0203	
SWEDISH KRONA TO US NOON NY	0.0073		0.0000		0.0015		0.0001		0.0004	
SWISS FRANC TO US $ NOON NY	0.0010		0.0001		0.0000		0.0001		0.0000	
AUSTRALIAN *TOUS* NOON NY	0.0094		0.0001		0.0012		0.0001		0.0001	
NEW ZEALAND *TOUS* NOON NY	0.0248		0.0000		0.0006		0.0001		0.0000	
UK £ TO US $ NOON NY	0.0211		0.0001		0.0016		0.0002		0.0003	
INDIAN RUPEE TO US $ NOON NY	0.0045		0.0001		0.0096		0.0002		0.0127	
SRI LANKAN RUPEE TO US $ NOON NY	0.0034		0.0002		0.0124		0.0002		0.0092	
CHINESE YUAN TO US $ NOON NY	0.0000		0.0009		0.0000		0.0002		0.0000	
HONG KONG *TOUS* NOON NY	0.0000		0.0158		0.0000		0.0002		0.0000	
SINGAPORE *TOUS* NOON NY	0.0000		0.0002		0.0000		0.0002		0.0000	
THAI BAHT TO US $ NOON NY	0.0243		0.0001		0.0030		0.0003		0.0000	
SOUTH KOREAN WON TO US $ NOON NY	0.0380		0.0000		0.0062		0.0005		0.0026	
TAIWAN NEW *TOUS* NOON NY	0.0000		0.0003		0.0000		0.9959		0.0000	

Table 7: Out-of-sample portfolio composition for metals

Entries report the optimal weights of the traditional portfolio and the augmented portfolio with metals. There are two portfolio types, namely the "traditional" and the augmented. The dataset spans the period 1/1/2000 - 4/30/2020, for a total of 224 monthly returns and the rolling window analysis covers 120 months (10 years).

	cpt augm	cpt trad	markowitz augm	markowitz trad	ra augm	ra trad	la augm	la trad	rn augm	rn trad
US TREASURY CONST MAT 30 YEAR	0.0000	0.0000	0.2241	0.2802	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
US TREASURY CONST MAT 5 YEAR	0.0000	0.0000	0.3008	0.2447	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
US CORP BONDS MOODYS SEASONED AAA	0.0000	0.0000	0.1882	0.2037	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
US CORP BONDS MOODYS SEASONED BAA	0.9079	0.8407	0.2846	0.2698	0.9424	0.9510	0.0002	0.0000	0.9376	0.9468
Bloomberg Barclays U.S. Aggregate USD	0.0008	0.0467	0.0014	0.0008	0.0000	0.0000	0.0003	0.0000	0.0000	0.0000
S&P500 ES HEALTH CARE	0.0094	0.0105	0.0001	0.0001	0.0058	0.0082	0.0003	0.0000	0.0068	0.0097
S&P500 ES CONSUMER DISCRETIONARY	0.0051	0.0012	0.0000	0.0001	0.0030	0.0024	0.0003	0.0000	0.0028	0.0019
S&P500 ES CONSUMER STAPLES	0.0091	0.0093	0.0002	0.0003	0.0073	0.0076	0.0004	0.0000	0.0113	0.0100
S&P500 ES INDUSTRIALS	0.0000	0.0002	0.0001	0.0001	0.0005	0.0009	0.0004	0.0000	0.0000	0.0001
S&P500 ES INFO TECHNOLOGY	0.0050	0.0084	0.0000	0.0001	0.0039	0.0052	0.0005	0.0001	0.0025	0.0018
S&P500 ES MATERIALS	0.0006	0.0112	0.0000	0.0000	0.0001	0.0019	0.0006	0.0001	0.0000	0.0003
S&P500 ES COMM. SVS	0.0124	0.0181	0.0000	0.0000	0.0025	0.0044	0.0006	0.0005	0.0008	0.0009
S&P500 ES UTILITIES	0.0068	0.0171	0.0000	0.0000	0.0081	0.0050	0.0008	0.0006	0.0096	0.0062
S&P500 ES FINANCIALS	0.0048	0.0125	0.0000	0.0000	0.0003	0.0005	0.0011	0.0010	0.0000	0.0000
S&P500 ES ENERGY	0.0055	0.0242	0.0000	0.0001	0.0066	0.0130	0.0015	0.9975	0.0067	0.0223
Palladium U$/Troy Ounce	0.0077		0.0000		0.0044		0.0018		0.0045	
London Platinum Free Market $/Troy oz	0.0024		0.0000		0.0035		0.0019		0.0042	
Gold, Handy & Harman Base $/Troy Oz	0.0108		0.0001		0.0097		0.0103		0.0122	
Silver, Handy&Harman (NY) U$/Troy OZ	0.0119		0.0000		0.0018		0.9784		0.0010	

Table 8: Scaillet & Topaloglou Non - parametric tests for SSD

The null hypothesis is that the Traditional portfolio SSD the Augmented, in every case. P-values are obtained through bootstrapping with overlapping blocks. We reject the null if the p-value is lower than 5%.

Significance level at 5%		p-value
Real Estate	CPT	0.0691
	Markowitz	0.0451
	Risk Averse	0.0537
	Loss Averse	0.0314
Commoditi es	CPT	0.0317
	Markowitz	0.0053
	Risk Averse	0.0859
	Loss Averse	0.0001
Foreign Exchange	CPT	0.0812
	Markowitz	0.0654
	Risk Averse	0.0728
	Loss Averse	0.0001
Precious Metals	CPT	0.021
	Markowitz	0.0526
	Risk Averse	0.0612
	Loss Averse	0.0261

Table 9: Experiment 1 (Real Estate) out-of-sample performance: Parametric portfolio measures

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss) for the traditional and the enhanced optimal portfolios. The results for the opportunity cost are reported for different degrees of absolute risk aversion (RA) (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6), as well as for CPT, Loss Averse (LA), Risk Neutral (RN) and Markowitz value functions. All values are rounded to the fifth decimal.

Performance measures	CPT (T)	CPT(E)	Mark. (T)	Mark. (E)	RA (T)	RA (E)	RN (T)	RN (E)	LA (T)	LA (E)
Sharpe ratio	0.46992	0.43011	0.16422	0.35035	0.73954	0.72004	0.32456	0.31290	0.23314	0.22317
Downside Sharpe .	0.15905	0.12734	0.12345	0.22873	0.34401	0.29756	0.31278	0.33425	0.17895	0.16997
UP ratio	0.99196	0.62420	0.91898	0.92661	1.12275	1.07888	0.87965	0.65355	0.49367	0.66324
Portfolio Turnover	0.10227	0.09403	1.57051	0.09403	0.07985	0.09403	0.05436	0.44529	0.95367	0.14794
Return Loss	0.06217	-	0.16767	-	0.06217	-	0.12657	-	0.16981	-
Opportunity cost *Exponential utility*	-0.00007	-	-0.00014	-	-	-	-	-	-0.00040	-
ARA=2					0.00008					
ARA=4					0.00009					
ARA=6					0.00009					
Power utility										
RRA=2					0.00018					
RRA=4					0.00036					
RRA=6					0.00054					

Table 10: Experiment 2 (Commodities) out-of-sample performance: Parametric portfolio measures

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss) for the traditional and the enhanced optimal portfolios. The results for the opportunity cost are reported for different degrees of absolute risk aversion (RA) (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6), as well as for CPT, Loss Averse (LA), Risk Neutral (RN) and Markowitz value functions. All values are rounded to the fifth decimal.

Performance measures	CPT (T)	CPT (E)	Mark. (T)	Mark. (E)	RA (T)	RA (E)	RN (T)	RN (E)	LA (T)	LA (E)
Sharpe ratio	0.18518	0.23241	0.07049	0.23494	0.76613	0.48647	0.08216	0.01865	0.15321	0.18452
Downside Sharpe	0.05459	0.04906	0.57683	0.68795	0.46396	0.13794	0.05525	0.44278	0.71133	0.29984
UP ratio	0.54667	0.89140	0.73909	0.82707	1.08020	0.87139	1.25450	0.74536	0.76660	0.89721
Portfolio Turnover	0.12227	0.10724	0.57051	1.12724	0.06953	0.02724	0.95365	0.76489	0.11761	0.10001
Return Loss	-0.09877	-	0.17200	-	0.08977	-	0.23723	-	0.07726	-
Opportunity cost	0.00022	-	0.00029	-	-	-	-	-	0.00013	-
Exponential utility										
ARA=2					-0.00094					
ARA=4					-0.00011					
ARA=6					-0.00013					
Power utility										
RRA=2					-0.00019					
RRA=4					-0.00035					
RRA=6					-0.00055					

Table 11: Experiment 3 (FX) out-of-sample performance: Parametric portfolio measures

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss) for the traditional and the enhanced optimal portfolios. The results for the opportunity cost are reported for different degrees of absolute risk aversion (RA) (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6), as well as for CPT, Loss Averse (LA), Risk Neutral (RN) and Markowitz value functions. All values are rounded to the fifth decimal.

Performance measures	CPT (T)	CPT (E)	Mark. (T)	Mark. (E)	RA (T)	RA (E)	RN (T)	RN (E)	LA (T)	LA (E)
Sharpe ratio	0.22377	0.19500	0.69690	0.58526	0.77321	0.58526	0.45360	0.23141	0.22315	0.19892
Downside Sharpe	0.07213	0.06660	0.07114	0.06372	0.46927	0.35578	0.21321	0.11324	0.00552	0.02289
UP ratio	0.57059	0.47439	0.82508	0.76874	0.97566	0.83015	0.47372	0.25784	0.09984	0.01698
Portfolio Turnover	0.18356	0.12003	1.45715	0.23323	0.70204	0.24876	0.45311	0.33201	0.07568	0.08569
Return Loss	-0.01760	-	0.17635	-	-0.01241	-	-0.00013	-	-0.01032	-
Opportunity cost	-0.00001	-	-0.00002	-	-	-	-	-	-0.00034	-
Exponential utility										
ARA=2					0.00006					
ARA=4					0.00005					
ARA=6					0.00004					
Power utility										
RRA=2					0.00005					
RRA=4					0.00002					
RRA=6					0.00001					

Table 12: Experiment 4 (Precious Metals) out-of-sample performance: Parametric portfolio measures

Entries report the performance measures (Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss) for the traditional and the enhanced optimal portfolios. The results for the opportunity cost are reported for different degrees of absolute risk aversion (RA) (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6), as well as for CPT, Loss Averse (LA), Risk Neutral (RN) and Markowitz value functions. All values are rounded to the fifth decimal.

Performance measures	CPT (T)	CPT (E)	Mark. (T)	Mark. (E)	RA (T)	RA (E)	RN (T)	RN (E)	LA (T)	LA (E)
Sharpe ratio	0.14234	0.16533	0.09929	0.09493	0.09493	0.56736	0.00879	0.15657	0.85841	0.91434
Downside Sharpe	0.06117	0.07854	0.08789	0.07277	0.30613	0.18017	0.45288	0.47324	0.05522	0.76809
UP ratio	0.54538	0.70352	0.86384	0.82802	1.06850	1.00160	0.09984	0.11523	0.43562	0.76459
Portfolio Turnover	0.01779	0.05364	1.48799	0.08680	0.07571	0.06674	0.65499	0.54881	0.08193	0.08475
Return Loss	-0.01326	-	0.16379	-	-0.01386	-	0.03425	-	0.32333	-
Opportunity cost	0.00004	-	-0.00002	-	-	-	-	-	0.00002	-
Exponential utility										
ARA=2					-0.00003					
ARA=4					-0.00005					
ARA=6					-0.00008					
Power utility										
RRA=2					-0.00001					
RRA=4					-0.00003					
RRA=6					-0.00004					

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